THE UNIVERSITY OF ALBERTA

UPPER MANTLE STRUCTURE IN WESTERN CANADA

by

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The undersigned certify that they have read,
and recommend to the Faculty of Graduate Studies for
acceptance, a thesis entitled UPPER MANTLE STRUCTURE
IN WESTERN CANADA submitted by Charles David Hemmings
in partial fulfilment of the requirements for the degree
of Master of Science.

ABSTRACT

Refraction arrivals from the upper mantle were recorded on a 3500 km. profile stretching from Lake Superior to Alaska. Recordings from the middle section of the profile (1000 - 2000 km.) were digitized, and various frequency and velocity filters were used to enhance this data. One version of the velocity filter, a non-linear operator, obtained the highest degree of data enhancement, but at the expense of signal distortion. The analysis presented in this paper rests primarily on these enhanced records and secondarily on the analog records.

The velocity structure was obtained by matching a theoretical travel time curve calculated from proposed structures to the observed travel time curve. The most successful model thus produced was model U of A #3. This model contained four striking features; two velocity reversals, one at a depth of 42 to 70 km. and the other at 125 to 170km., both of which were followed by rapid velocity increases, and two rapid velocity increases at depths of 455 and 660 km. These velocity reversals were necessary to reproduce the shadow zone at a distance of 900 km. and its accompanying cusp at 300 km., and the shadow zone at 1500 km. with its cusp at 800 km. The rapid velocity increases were used to produce two cusp

pairs, the first at 3200 and 1700 km., and the second at
3300 and 2200 km. Two additional velocity reversals
at depths of 198 to 230 km. and 250 to 325 km. with a
small velocity increase between them were neede to
produce the shadow zones at 2000 and 2400 km. respectively.
These reversals were too small to produce any detectable
cusping.

To complete the details of this structure,
and to obtain the velocity behaviour in the transition
zones, the theoretical amplitudes calculated from the
model must be made to match the observed amplitudes.
This amplitude study is not presented in this paper.
However, the computer program listed in the appendix
calculates both amplitudes and travel times, and is
presently being used to complete this study.

ACKNOWLEDGEMENTS

I take great pleasure to have this opportunity to thank Dr. E. R. Kanasewich for the constant encouragement and guidance provided by him throughout my entire course of study at the University of Alberta.

I would also like to thank Dr. G. L. Cumming for his work during the initial stages of Project Early Rise.

A special vote of thanks to the technicians and field personnel, from both the University of Alberta and the United States Geological Survey, for their diligence in recording and digitizing the data, which is the backbone of any seismic research.

I would like to thank the University of Lethbridge for their help in printing this paper, particularly Mr. Boon.

Finally, I would like to dedicate this thesis to my wife, Betti. Without her constant understanding and support this thesis would not have been completed. Also, I wish to acknowledge the work she has done on this paper, which involved, among other things, the proof-reading and typing of the manuscript.

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CHAPTER 1.

DATA COLLECTION

1.1   Introduction

Project Early Rise, sponsored by the United
States Air Force Office of Scientific Research and co-
ordinated by the United States Geological Survey, was
an experiment designed for the investigation of the
upper mantle using seismic body waves. In previous
studies, earthquakes and nuclear explosions were used as
the seismic source. However, due to the nature of these
sources, the resulting refraction profiles were often in
anomalous areas and were necessarily short or sparsely
occupied. Also, the source parameters, such as origin
time, hypocenter, energy, and transfer function, were not
always well known, which added further confusion to the
complex travel time and amplitude curves for seismic body
waves. For a successful experiment, the profiles had to
be long and detailed, with as many of the source parameters
independently measurable or known as possible. Project
Early Rise was planned to meet these requirements.

1.2   Planning

Probably never before had such a wealth of

information been available for the planning of a refraction program. Cumming and Kanasewich (1965) demonstrated the feasibility of using underwater chemical explosions of a reasonable size as the seismic source for upper mantle investigation. They recorded several of the two to ten ton shots from the 1963 Lake Superior crustal experiment at distances sufficiently great (1600 - 1900 km.) that the seismic waves producing the first events had penetrated the mantle. This experiment also provided a known crust (Berry and West, 1966) under Lake Superior and provided data regarding seismic coupling of underwater explosions at various locations within the lake. Underwater shots had a second advantage due to the fact that all shots could be set off at the same location, thus yielding a common transfer function, coupling, and crust for all explosions. The same experiment provided valuable experience in shot procedure. Most important, it resulted in a proven system of hydrophone stations for measuring the shot instant and precise shot location. The 1963 experiment, as well as another one in 1964, guaranteed a reliable source with accurately measurable source parameters.

As a result of these studies, forty shots of five tons each were detonated at a common shot point in July 1966 during Project Early Rise. These formed the sources for the many seismic profiles, shown in figure 1.1, under-taken by various university and government research

Figure 1.1   Project Early Rise.
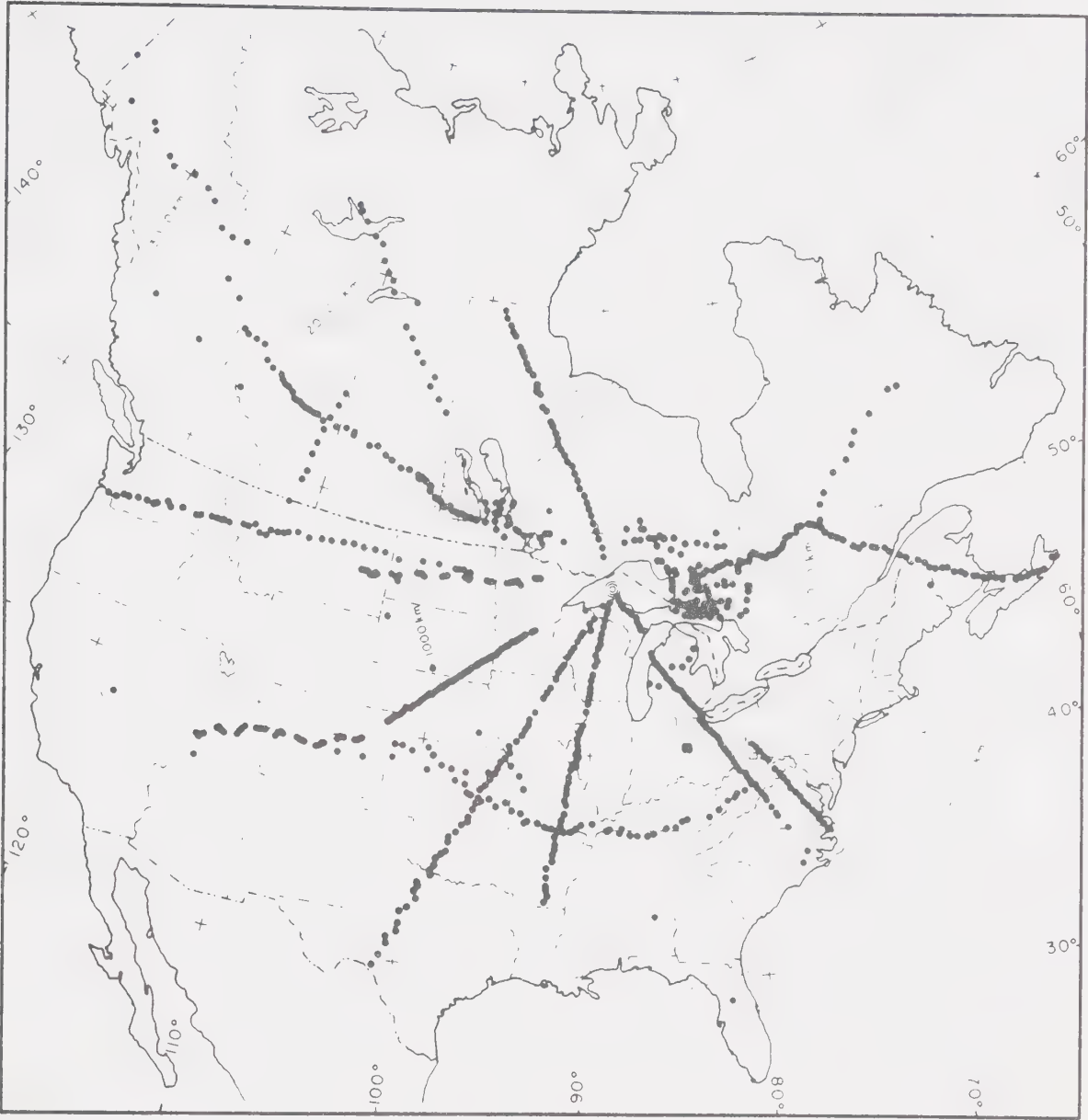
laboratories. Five seismic crews, two from the University
of Alberta and three from the United States Geological
Survey (USGS), employing mobile linear arrays, participated
in the Western Canada (WC) profile, consisting of 74
recording sites running along a WNW line (figure 1.2).
The first site was 400 km. from the shot point and the
last 3600 km. In addition, 9 sites made up an arc 1675 km.
from the shot point. Stretching from Cold Lake, Alberta,
to just south of the Trans Canada Highway near Suffield,
Alberta, this arc encompassed approximately 20 degrees.
Generally, the spacings between sites was 50 km., which
was increased beyond the range of 2000 km. It was felt
that this spacing would preserve the signal character from
one site to the next, and this character could then be
used to trace events across the profile. Near Holden,
Alberta, from 1754 km. to 1781 km., the spacing was
reduced to the length of the array (2 miles). This
continuous section of the profile was to provide data
over an anticipated critical distance for arrivals from
below a possible low velocity layer in the mantle, and was
to be used for experiments in beam forming. In addition,
two permanent sites were occupied for the duration of the
experiment, one at the University of Alberta Edmonton
Seismic Observatory near Leduc, Alberta (1840 km.), and
the other at the intersection of the profile and the arc
(WC 36). These stations were expected to supply data for
stacking and for calibrating the spectral density estimates

Figure 1.2   The Western Canada profile.

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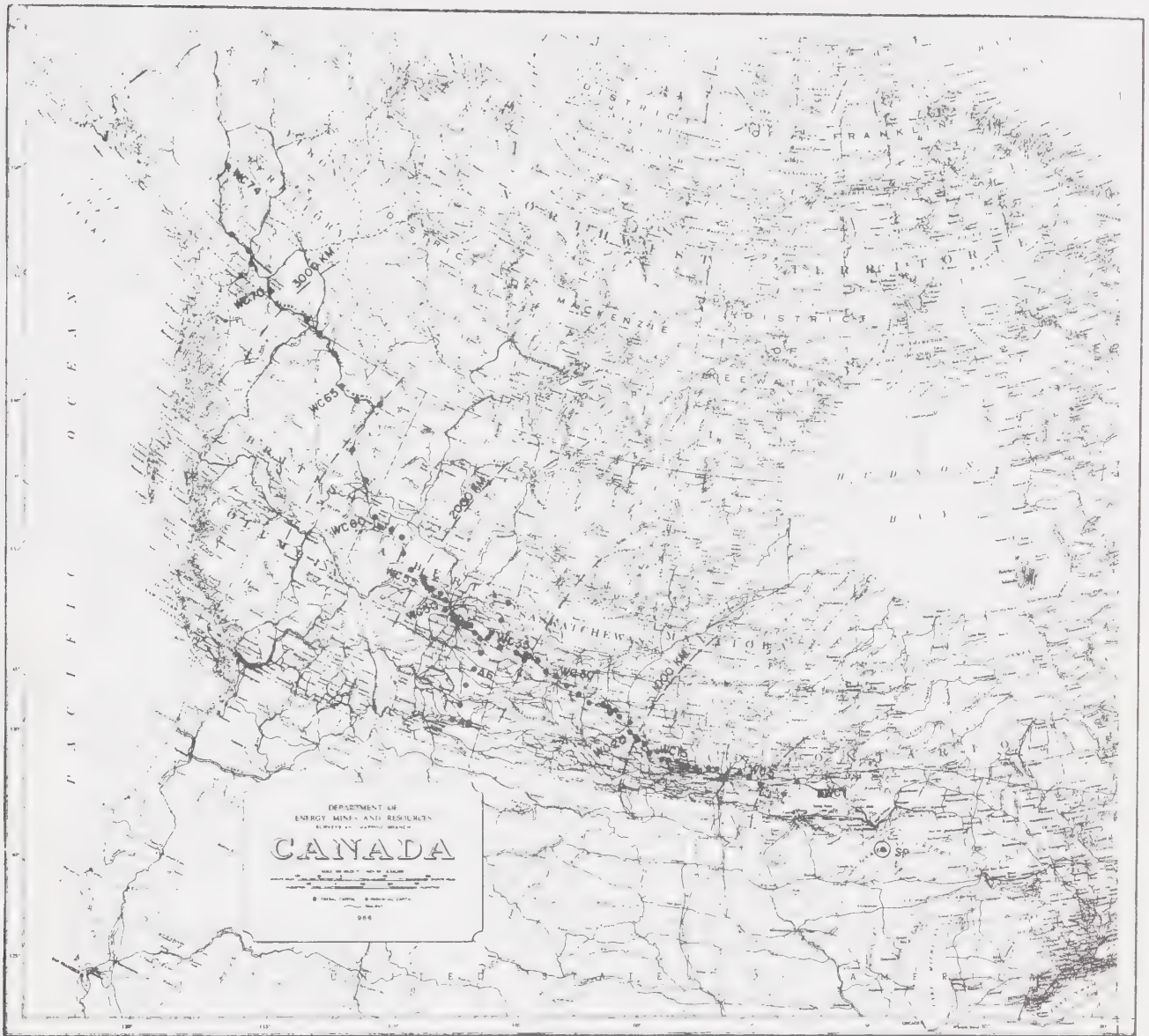
of the individual sources.

The proposed recording sites were taken from large scale maps (4 miles = 1 inch) and located on isolated EW roads. It was the responsibility of the recording crews to make the final selection of the recording locations when they arrived on the proposed site on the day it was to be occupied. In choosing the final site locations, the crews had to consider such features as power lines and seismic background noise, while maintaining the proposed spacings on the profile. Since these were small, it was considered best if each crew worked a particular range of the profile. In this way they could consider the final location of the previously occupied site in choosing the new one, thus preserving the planned spacings. The crews also had to estimate the recording times at each location. Initially they used a velocity of 8.2 km./sec. and an intercept time of +5 seconds to estimate the arrival times, as taken from a review of the Cumming and Kanasewich data. These times were to be modified and the recording gains adjusted by extrapolating the data collected during the exercise. On the other hand, crews occupying adjacent sites would have more information on which to base these decisions, and an experienced observer could review all the records and make these decisions while the crews slept. Also, one technician could then service all of the recording equipment. Ideally, of course, these two

specialists would have no recording responsibilities.

Both systems were employed. The three USGS crews occupied adjacent sites on the close range section of the profile (400 to 1000 km.), after which each crew took on profile assignments on the far section (2000 to 3600 km.) The University of Alberta crews were given profile assignments on the central section of the profile (1000 to 2000 km. including the arc). Both systems worked well. It was found that the freedom of site selection offered by the profile section assignments was extremely valuable in the selection of good sites. However, due to the quality of data and the lack of time for adequate analysis, weak first arrivals were not always observed by the field parties, and hence no allowance was made for recording them on subsequent days. This type of oversight proved a handicap in the final data analysis, and could possibly have been avoided had the adjacent site method been used.

1.3   Field Equipment

The USGS crews employed their standard mobile 6 station linear array refraction equipment. The University of Alberta crews used their mobile 12 station refraction / reflection linear array apparatus, the same as used by Maureau (1964) during his crustal refraction program and by Clowes (1966, 1969) during his deep crustal reflection programs. The latter arrays consited of 12

stations with spacings of 960 feet covering 2 miles. At
station 6, as numbered from one end of the array, the leads
of stations 2 and 4 were tapped, and a three component
(vertical, radial, and transverse) station was established
using Hall-Sears HS-10 seismometers. Elsewhere, other
than at 2 and 4, Texas Instruments vertical S-36 seismo-
meters were employed. See figures 1.3 and 1.4 for the
seismometer responses. All components of the array, in
addition to timing information, were recorded on a RS-8U
recording oscillograph. These records will hereafter be
referred to as the field monitor records. A Precision
Instrument PS 207 AM/FM seven channel tape deck was used
to record 6 components of the array in FM, and one channel
of timing information in AM. Two modes of recording were
available, one recording the odd numbered array components,
hereafter referred to as the normal mode, and another,
the reverse mode, recording the even components. Thus
either a 6 station or a 4 station (having in addition the
two horizontal components) linear array, both with spacings
of 1920 feet, were available for replay and digitization
in the Edmonton laboratory.

1.4   Field Operation

The field operation, then, entailed the location
of a noise-free site, setting up the array and the recor-
ding equipment, and recording the shot. Noise records were

Figure 1.3   Frequency response of the Hall-Sears HS-10

seismometers.

HALL-SEARS, INC.

TYPE HS-10 SEISMIC DETECTOR
OUTPUT VS FREQUENCY

MODEL - K
NATURAL FREQUENCY - 20 CPS
COIL RESISTANCE - 4100 OHMS

LOAD	DAMPING
1. OPEN CIRCUIT	19%
2. 51 K	32%
3. 24 K	45%
4. 5 K	56%
5. 10 K	71%
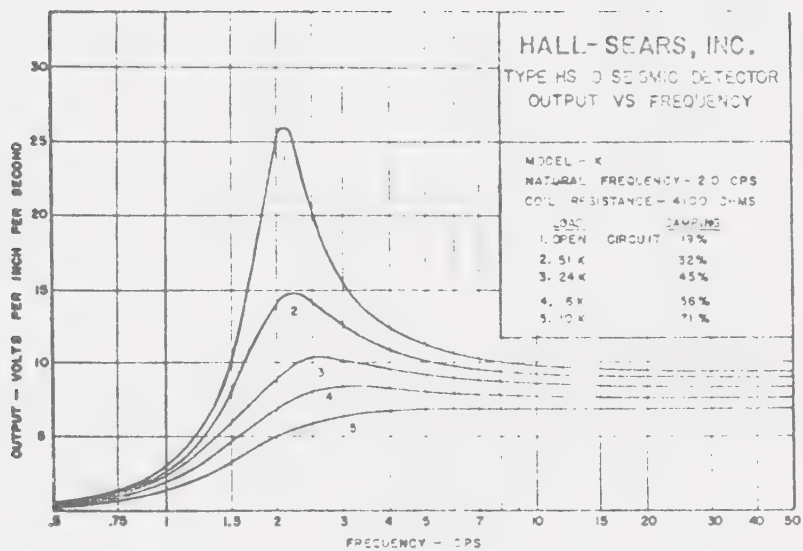
Figure 1.4   Frequency response of the Texas Instruments
vertical S-36 seismometers.

TI S-36

NATURAL   FREQUENCY — 2 CPS

COIL   RESISTANCE — 4K OHMS

	LOAD	DAMPING
1.	INFINITE	0
2.	15 K	.311
3.	6 K	.500
4.	2.6 K	.707
5.	444	1.00
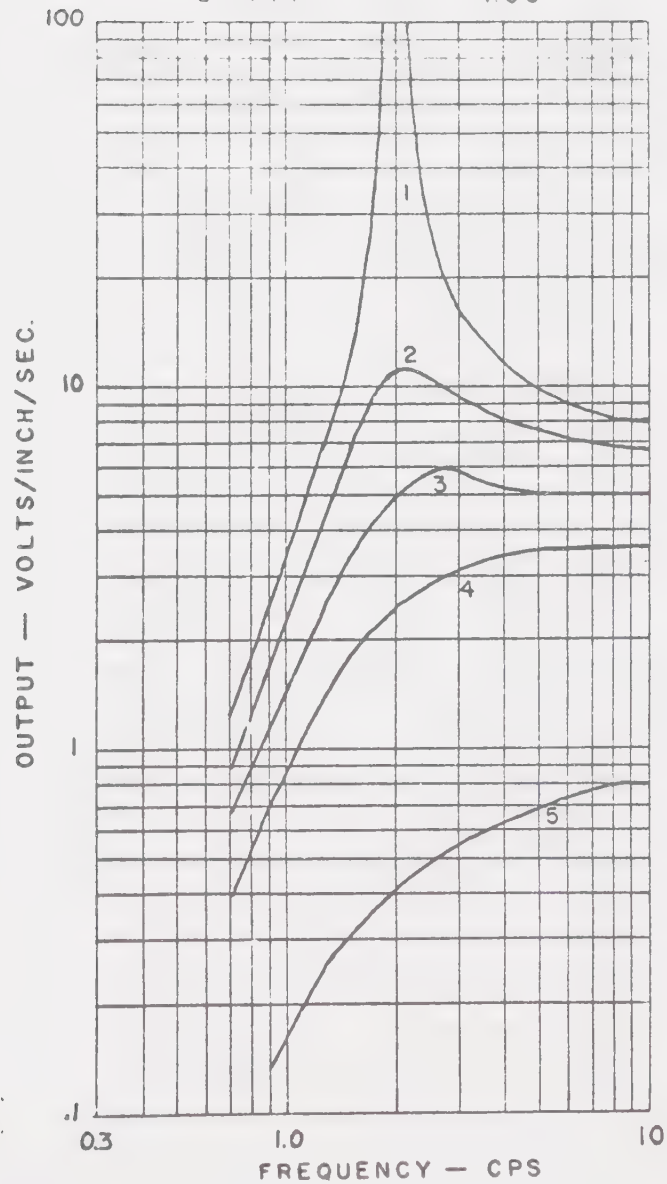
run to determine the state of the system and the background noise level at different filter frequencies. These recordings were run several times. Once before the system was completely set up, leaving ample time for repairs or site relocation, and again as soon as the station was in the ready state, in order to determine attenuations and filter settings for the recording of the shot. The first shot was recorded in the normal mode, using the data collected at previously occupied sites as a guide for recording times and attenuations. The record quality and the arrival times of this record were then checked on the field monitor record and used as a guide to record the second shot of the day, which was recorded at the same site. The second shot could be recorded in the reverse mode by reversing the array cable inputs; hence the name, reverse mode. Top priority was placed on the normal mode recording, as it provided the largest array on tape. If the first shot record was of good quality, the second shot of the day, at alternate sites only, was recorded in the reverse mode. A day's operation consisted of the recording of two shots on both tape and photographic paper.

1.5   Digitization

The digitizing system consisted of a 6 channel hot wire recorder for monitoring the tape channels, a

two channel pen recorder for the tape timing channel and
the digitizing time, aliasing filters, a 6 channel Packard
Bell multiplexer, a Packard Bell 11 bit analog to digital
converter, a clock (a square wave oscillator which governed
the conversion rate), a digital counter, switching circuit
between two Kennedy digital tape recorders, and the PS 207
tape deck in the replay mode. Because the system was not
designed to convert and record seismic data rapidly
enough (over 100 samples per second (s/s) per channel
were required for statistical significance, requiring
600 s/s by the system), the FM tape which was recorded at
30 inches per second (i/s) was replayed at $3\frac{3}{4}$ i/s.
Thus the digitizing time was increased a factor of 8
over real time, and a conversion rate of 90 s/s digitizing
time was equivalent to 120 s/s per channel real time,
which slightly exceeded the required rate. The aliasing
filter, 24 db per octave, corner frequency of 4.0 HZ
digitizing time was used to assure that there was no
signal of any significant power at a frequency higher than
the aliasing frequency (7.5 HZ machine time). It should
be noted that the data was recorded using a 24 db per
octave filter with a corner frequency of 8 to 16 HZ which
corresponds to 1 HZ to 2 HZ in digitizing time. Thus any
signal near or above the aliasing frequency was severely
attenuated.

The data was digitized in channel sequence with

a flag marker added to the word of the sixth channel, and was recorded on the digital tape in this form. This was organized into records consisting of 10 blocks of 3600 words each, giving a total of 36000 words per record or 6000 words per channel (just over 50 seconds real time of seismic data). Inter-record gaps are required between blocks. As the Kennedy recorders demanded more time to insert these than the sampling interval, two recorders and a switching circuit were used. When one recorder had recorded a full block, it put in its inter-record gap while the other recorder recorded the next block. This solution was considerably less expensive than using a high speed memory bank interfacing system. Thus each FM tape was digitized and recorded on two digital tapes.

The data was taken in this form to the IBM 360 Model 67 computer, and one tape containing all the data was built. During this process, longitudinal and horizontal parity checks and channel sequence checks, along with other IBM system checks, were made to insure that the digital data was reliable. On detection of an error the data was rejected and later redigitized. In this manner 114 digital records were compiled on two master tapes.

1.6   Bookkeeping and Data Limitations

Before data analysis could begin, a considerable amount of bookkeeping had to be done. The locations of the

shot points were taken from a technical letter published
by the United States Geological Survey (Warren, Healy,
Hoffman, Kempe, Rauula, and Stuart; 1967), and the recording
sites were determined from large scale maps. From this,
the profile distances were calculated using a Fortran
program based on a Clarke spheroid model of the earth.
The accuracy of the distances was limited to the scale of
the map and was estimated to be 0.2 km. Time was synchro-
nized between the shot point and the recording crews to an
accuracy of 0.005 second using WWV as a standard. Irre-
gularities in the spreads were noted in the field log
books, and the spread configurations calculated to the
nearest 2 meters. The digital data was plotted on the
CalComp model 663 plotter using the same time and amplitude
scales as those of the field monitor records. From a
direct comparison of the two, the time of beginning and
end of digitization was measured and the sampling interval
calculated. Using this data, the timing of events could
begin.

CHAPTER 2.

DATA ENHANCEMENT

2.1   Introduction

Any signal consists of two parts, one being wanted and the other unwanted, the latter being defined as noise. The noise can be broken down into various categories, depending on its characteristics; in fact, the noise signal can be considered as a combination of several types. The superposition theorem tells us that we can treat the wanted signal and each type of noise as separate signals. Signal enhancement, then, consists of the separation of these components, or at least the suppression of the noise.

Various processes have been derived for signal detection and noise rejection. These methods fall under two main headings, one being analog and the other digital. In general, for every analog process there exists an equivalent digital one, and vice versa. Thus this is not a true classification of procedures, but a classification of signal format and of the machine used to execute the procedure.

It becomes a question of which machine to use. Analog computers require separate electronic components

for each operation, such as addition, multiplication, delays, etcetera, and wires for controlling the operation sequence. The machines tend to be expensive and very specialized, with more noise being added to the signal by each amplifier. In digital computers, all operations are broken down into binary operations and the logic is represented by machine instructions generated by a language such as Fortran. Thus one machine can perform all the operations, with the programmer varying the logic as required. Hence the digital computer is extremely versatile and free of noise. The only noise introduced is in the round off of numbers during the binary operations, which may be rendered insignificant. However, because of the flexibility of the computer and the break-down of operations into simpler binary operations, the digital process can be substantially slower than the equivalent analog one. The selection of processes often breaks down into speed versus flexibility. For research purposes, flexibility is far more important, hence the analog signal was digitized and then processed on an IBM 360/67 computer.

2.2   Detection and Elimination of Spurious Transients

Transients or spikes are characterized by their rapid change to a maximum followed by an over-damped return to the normal signal level. These spikes occurred during field recording or during laboratory digitization. A

first difference method was used to detect the rapid
change and a linear interpolation to replace the spike.
A more sophisticated correction algorithm was not attempted
due to the difficulty in establishing a suitable detection
criterion. Moderate success was attained using the
deviation of the first difference from the mean first
difference as established over a two second interval in
the record, this window being moved across the entire
record. The allowable deviation was based on the standard
deviation from the first difference. Once the spike is
detected and its amplitude determined, and knowing the
time constant and the damping of the system which generated
the transient, it should be possible to calculate the
transient signal. Thus it can be removed by subtracting
the calculated signal from the seismic signal. It appears
that it is possible to write an anti-spike algorithm,
but the limitation of time prevented its full development.

2.3   Frequency Filtering

Local cultural activity and the wind generated
background noise which was generally of a higher frequency
than the wanted signal. A Butterworth zero phase band-pass
filter, programmed by Alpaslan (1968), was used to reject
this noise component. One filter with cut-off frequencies
of 1.0 and 5.0 HZ passed nearly all of the signal and
attenuated most of the noise. Another filter (0.5 and 3.0

HZ) slightly attenuated the signal and rejected almost all of the noise; however, it also removed the character of the signal. Figures 2.1 to 2.4 show the impulse and amplitude responses of these filters. Figure 2.5 shows some unfiltered Early Rise records, while figures 2.6 and 2.7 show the large improvement after application of these filters.

2.4   Linear and Non-Linear Beamforming

As this background noise was also incoherent, a method of beamforming which is based on this criterion for rejection is the simple delay and add routine. This is a linear method and amounts to timing the antenna (array) to point in a particular direction. The acceptance lobe can be sharpened by using weighting coefficients on each of the signals to be added. This procedure is commonly known as stacking or velocity filtering, as the delays depend on the geophone spacings and the assumed apparent velocity of the wanted arrival. Figure 2.8 shows the results of the before-mentioned records after application of this velocity filter.

Another beamforming technique employed is a non-linear variation of the delay and sum. This method (Muirhead, 1968) consists of a delay, taking the Nth root of the signal, summing, and then raising the result to the Nth power. Hereafter it will be referred to as the Nth root stack. As can be seen in figure 2.9, this technique had

Figure 2.1   Impulse response in the time domain for a
              four pole zero phase Butterworth 1.0 - 5.0
              HZ band pass filter.

IMPULSE RESPONSE OF BUTTERWORTH BAND PASS FILTER

IN PUT PARAMETERS

M=1   LCF= 1.0

T= 0.0063   HCF= 5.0

N= 1024.0
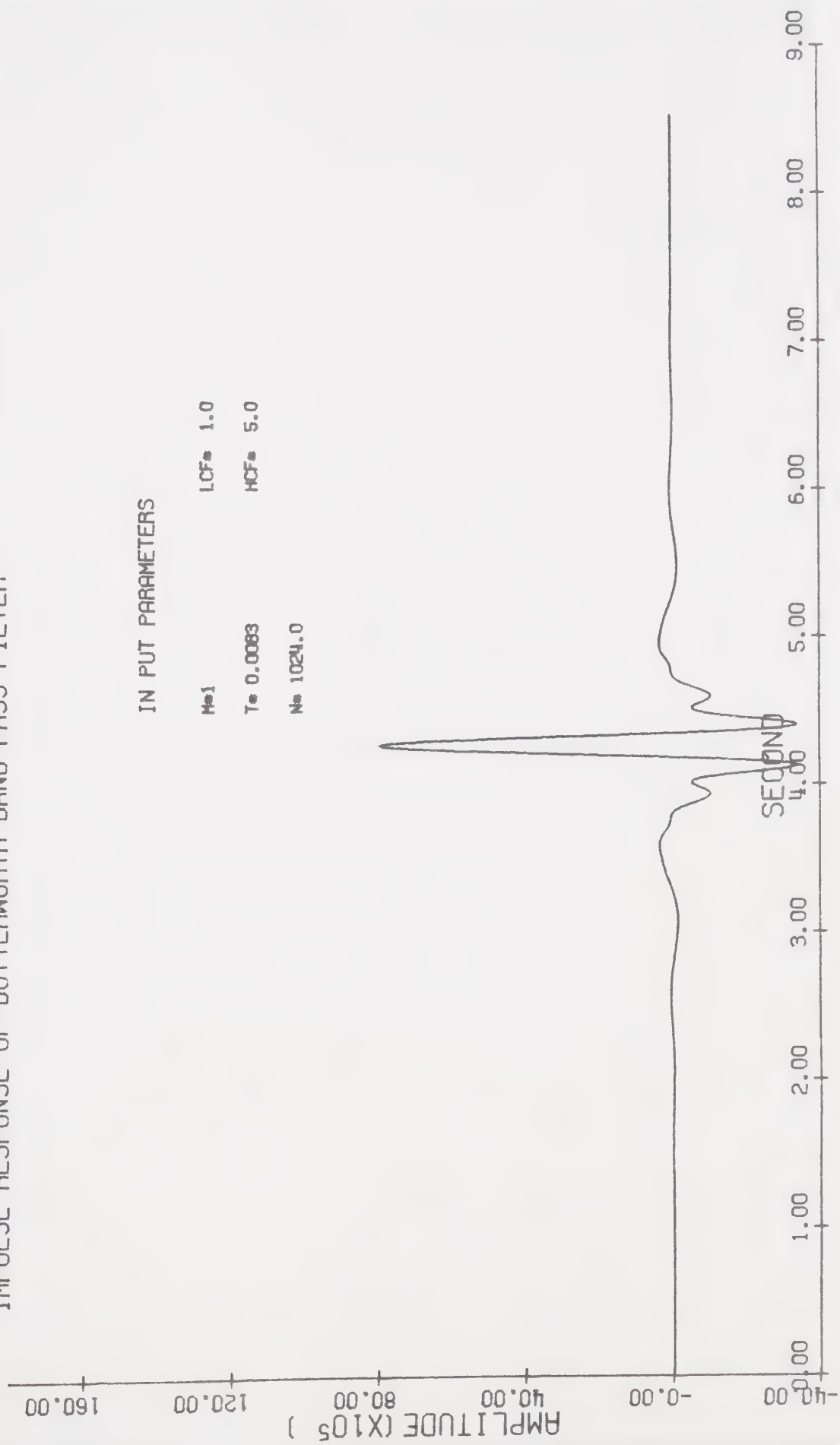
Figure 2.2   Impulse response in the frequency domain

for a four pole zero phase Butterworth

1.0 - 5.0 HZ band pass filter.

AMPLITUDE RESPONSE OF BUTTERWORTH BAND PASS FILTER FOR 1.0 - 5.0 HZ
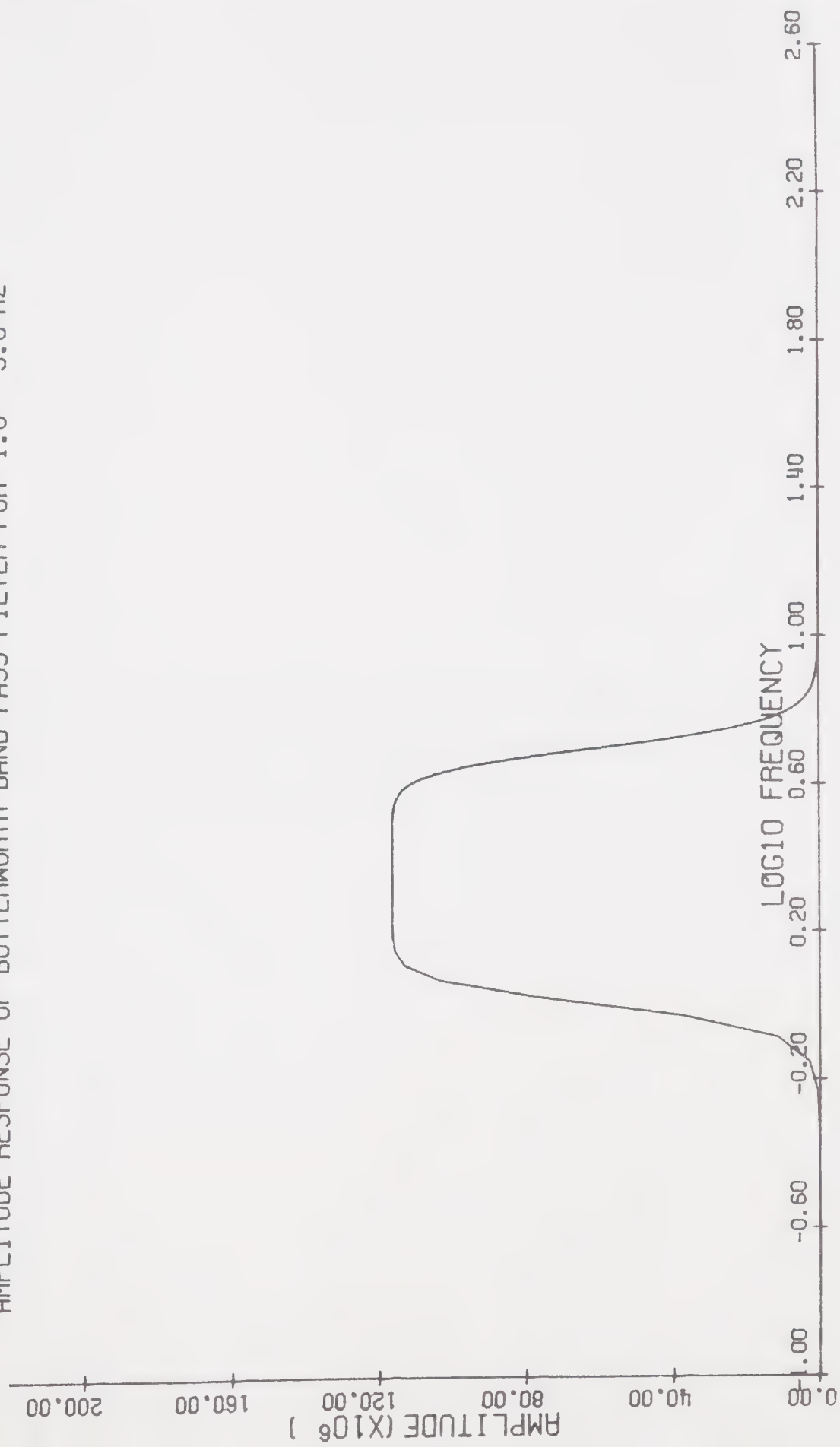
Figure 2.3   Impulse response in the time domain for a

four pole zero phase Butterworth 0.5 - 3.0

HZ band pass filter.

IMPULSE RESPONSE OF BUTTERWORTH BAND PASS FILTER

IN PUT PARAMETERS

M=1   LCF= 0.5

T= 0.0083   HCF= 3.0

N= 1024.0
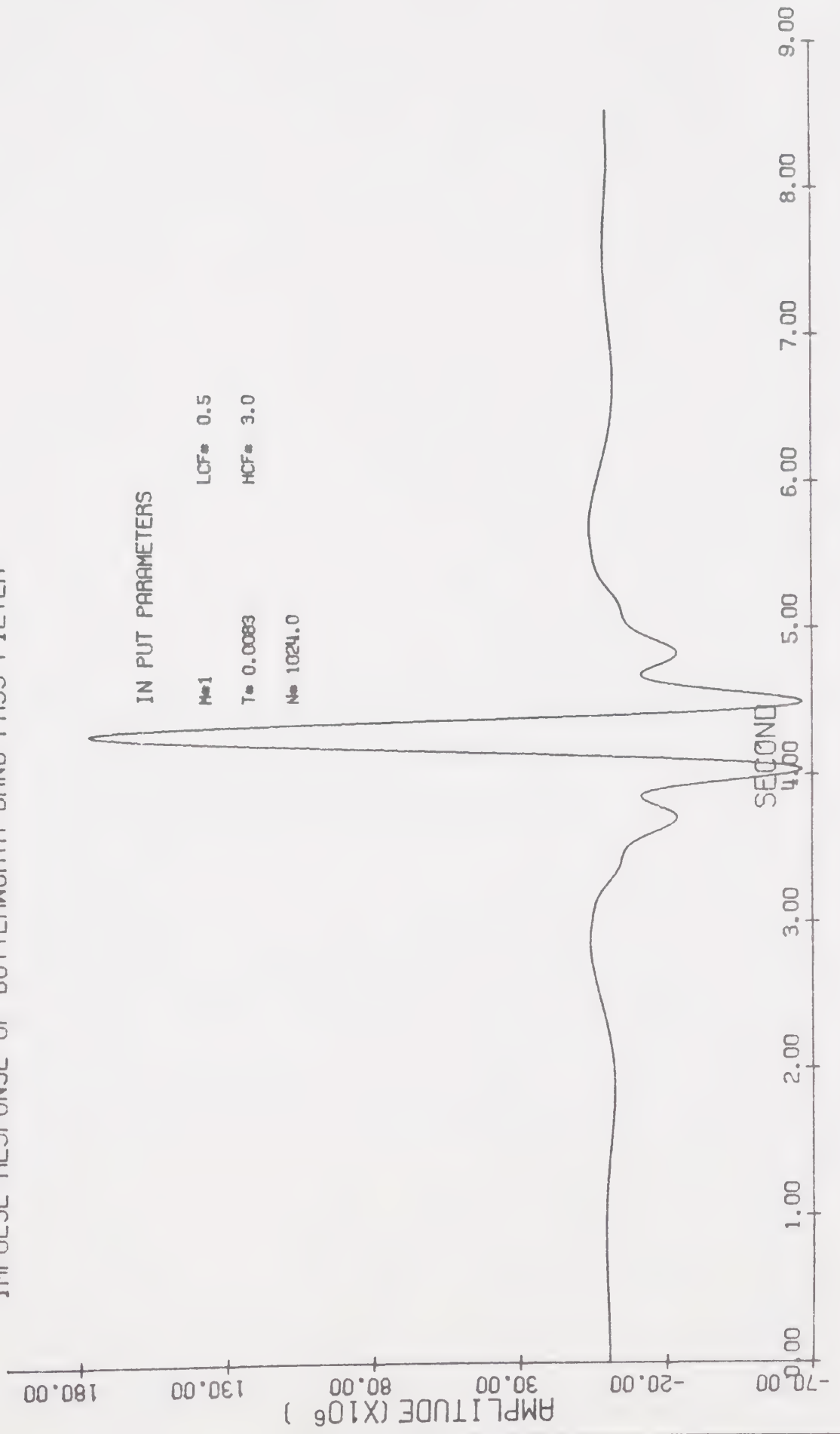
Figure 2.4   Impulse response in the frequency domain
             for a four pole zero phase Butterworth
             0.5 - 3.0 HZ band pass filter.

AMPLITUDE RESPONSE OF BUTTERWORTH BAND PASS FILTER FOR 0.5 - 3.0 HZ
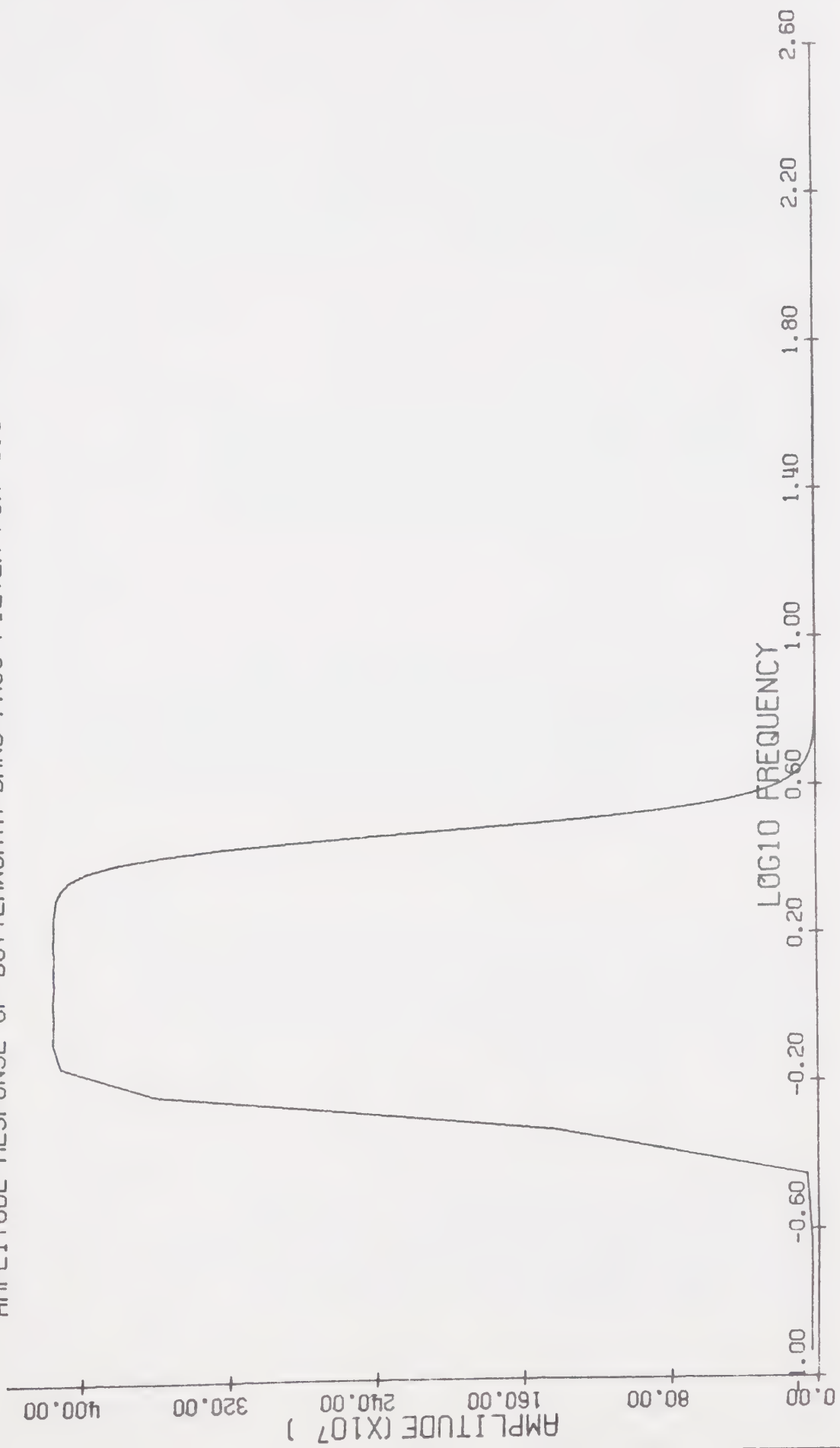
Figure 2.5   Five records covering the range 1128 to
1472 km. before data enhancement.

UNFILTERED T-DELTA/8.5

1128 KM.

RECORD 2
WC 25.0 SHOT 4.0
TODIG 2.82 SEC
DELTA 1128.39 KM
ATTN 18.0 DB

T-DELTA/8.5
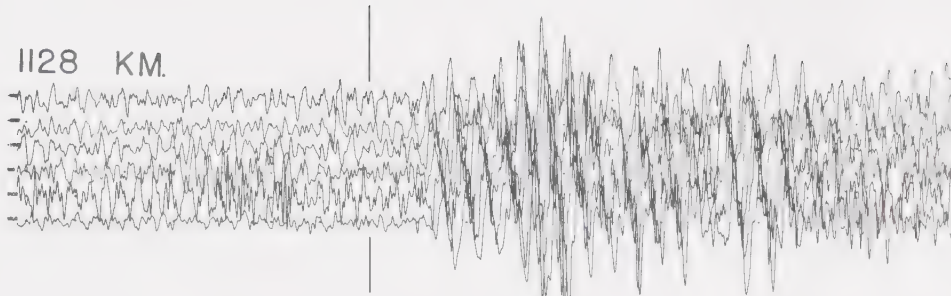
1140 KM.

RECORD 1
WC 24.0 SHOT 3.0
TODIG 5.21 SEC
DELTA 1140.40 KM
ATTN 18.0 DB
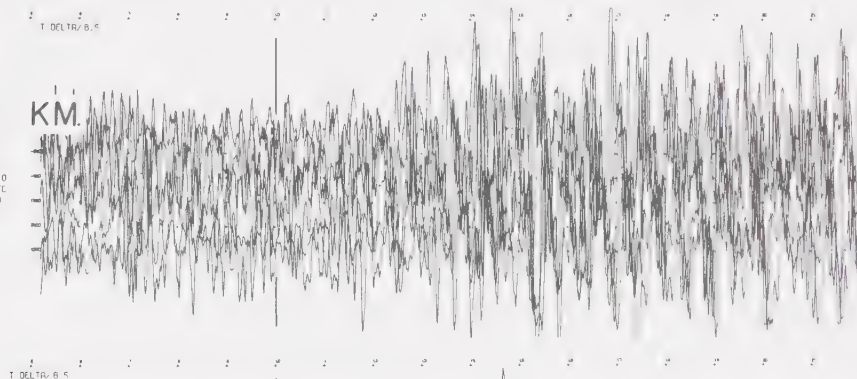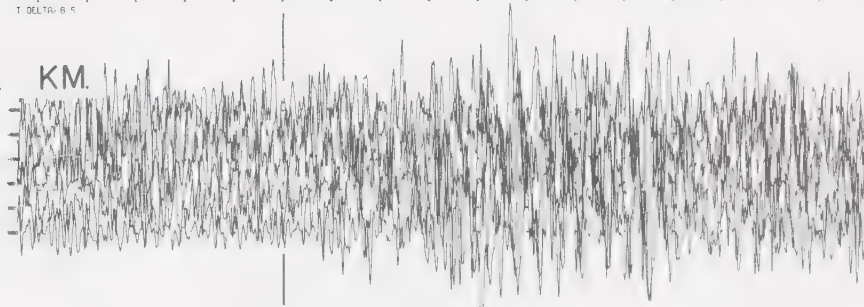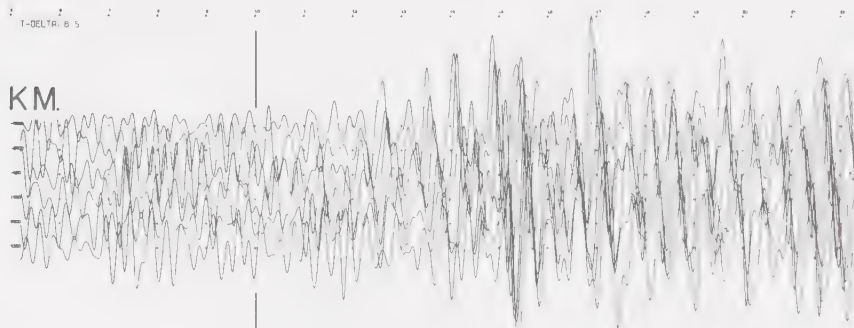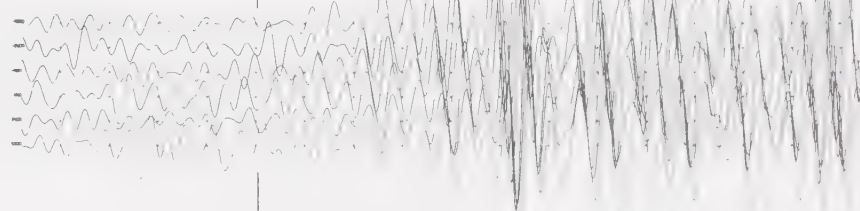
T-DELTA/8.5

1232 KM.

RECORD 5
WC 27.0 SHOT 6.0
TODIG 4.57 SEC
DELTA 1232.20 KM
ATTN 18.0 DB

T-DELTA/8.5

1428 KM.

RECORD 3
WC 31.0 SHOT 14.0
TODIG 4.25 SEC
DELTA 1428.47 KM
ATTN 18.0 DB
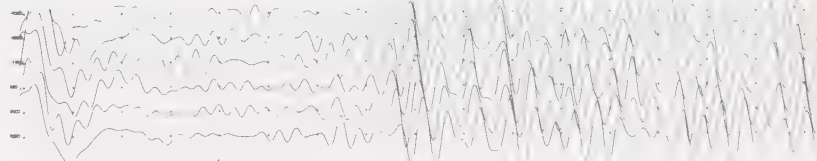
T-DELTA/8.5

1472 KM.

RECORD 15
WC 32.0 SHOT 16.0
TODIG 6.45 SEC
DELTA 1472.73 KM
ATTN 18.0 DB

Figure 2.6   These same records after applying the
              1.0 - 5.0 HZ Butterworth filter.

BANDPASS
1.0 - 5.0 HZ.

1128 KM.

1140 KM.

1232 KM.

1428 KM.

1472 KM.

Figure 2.7   These same records after applying the
              0.5 - 3.0 HZ Butterworth filter.

BANDPASS
0.5 - 3.0 HZ.

1128 KM.

RECORD 2
WC 25.0 SHOT 4.0
TODIG 2.82 SEC
DELTA 1128.39 KM
ATTN 18.0 DB
FIL(0.5 3.0)

1140 KM.

RECORD 1
WC 24.0 SHOT 3.0
TODIG 5.21 SEC
DELTA 140.43 KM
ATTN 18.0 DB
FIL(0.5 3.0)

1232 KM.

RECORD 5
WC 27.0 SHOT 6.0
TODIG 4.57 SEC
DELTA 1232.20 KM
ATTN 18.0 DB
FIL(0.5 3.0)

1428 KM.

RECORD 3
WC 31.0 SHOT 14.0
TODIG 4.25 SEC
DELTA 1428.47 KM
ATTN 18.0 DB
FIL(0.5 3.0)

1472 KM.

RECORD 15
WC 32.0 SHOT 16.0
TODIG 6.45 SEC
DELTA 1472.73 KM
ATTN 18.0 DB
FIL(0.5 3.0)

Figure 2.8   These same records after applying the
velocity filter.

VELOCITY
FILTER   1128 KM.   10 S.   20 S.

RECORD 2   0 MT STACK
WC 26.0   8.0 KM/SEC
SHOT 4.0   10.0 KM/SEC
1.00   5.0 HZ   12.0 KM/SEC
             14.0 KM/SEC
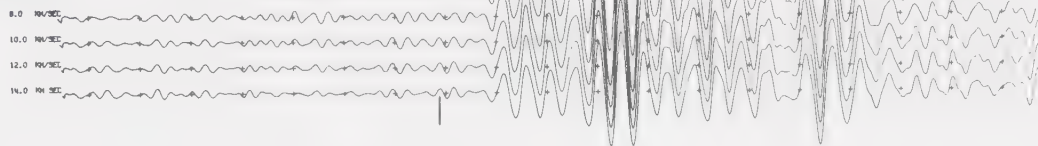
1140 KM.

RECORD 1   0 MT STACK
WC 24.0   8.0 KM/SEC
SHOT 3.0   8.0 KM/SEC
1.00   5.0 HZ   10.0 KM/SEC
             1.0 KM/SEC
             14.0 KM/SEC

1232 KM

RECORD 5   0 MT STACK
WC 27.0   6.0 KM/SEC
SHOT 8.0   8.0 KM/SEC
1.00   5.0 HZ   10.0 KM/SEC
             12.0 KM/SEC
             14.0 KM/SEC

1428 KM.

RECORD 9   0 MT STACK
WC 31.0   6.0 KM/SEC
SHOT 14.0   8.0 KM/SEC
1.00   5.0 HZ   10.0 KM/SEC
             12.0 KM/SEC
             14.0 KM/SEC

1472 KM.

RECORD 15 0 MT STA N
WC 32.0   6.0 KM/SEC
SHOT 16.0   8.0 KM/SEC
1.00   5. HZ   10.0 KM/SEC
             12.0 KM/SEC
             14.0 KM/SEC
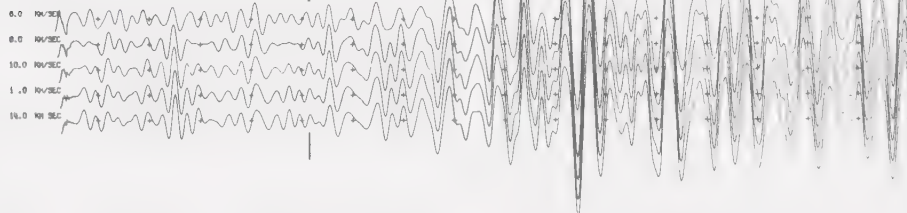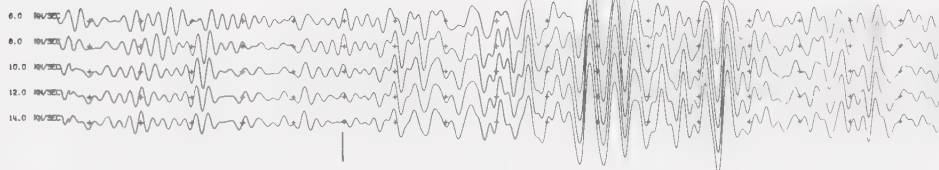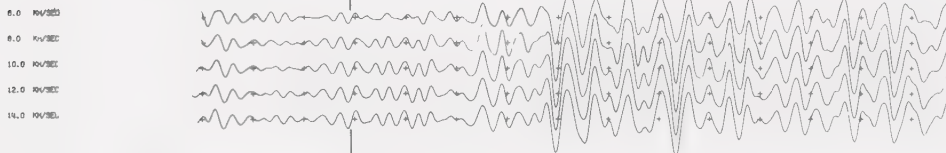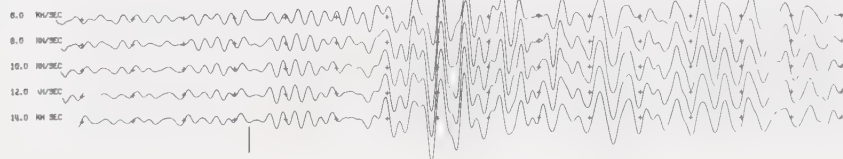
Figure 2.9   These same records after applying the
8th root stack filter.

8 ROOT STACK   1128 KM.   10 S.   20 S.
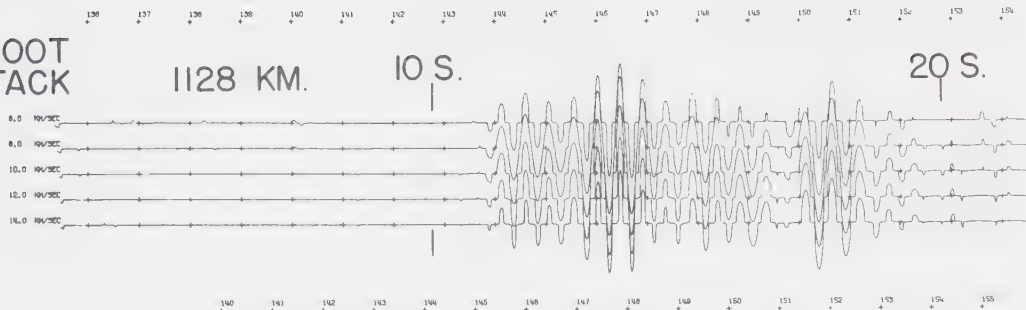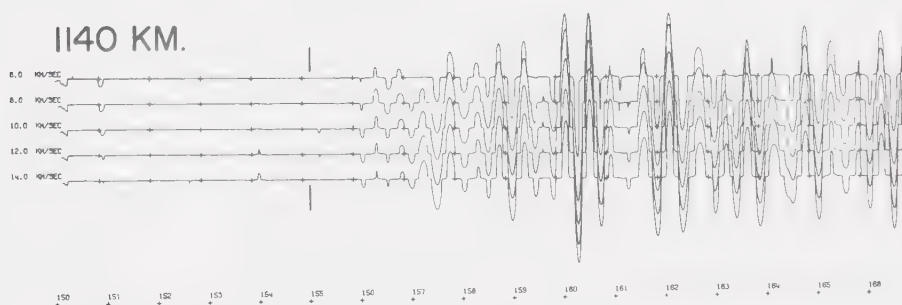
1140 KM.

1232 KM.
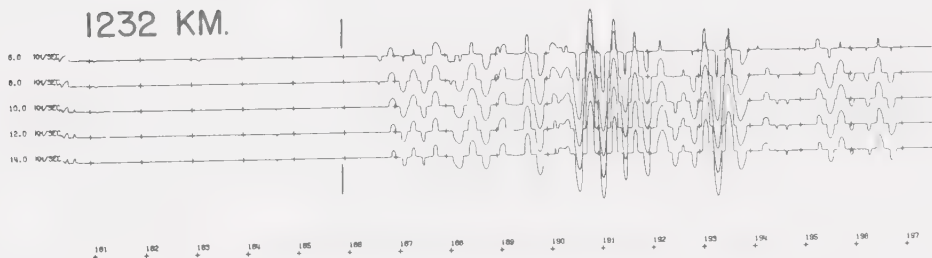
1428 KM.
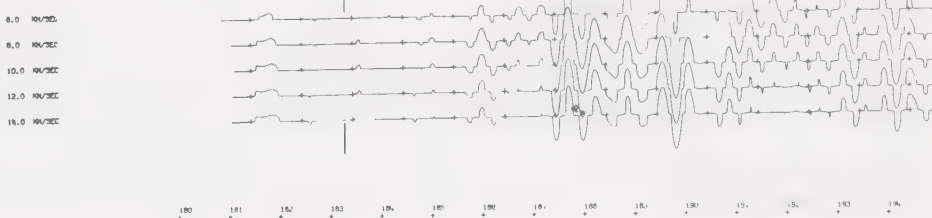
1472 KM.
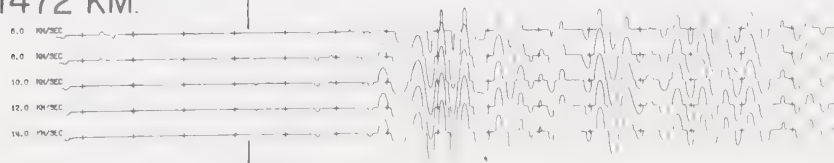
greater noise rejecting ability than the normal stack, and also had better velocity resolution. It achieves this at the expense of signal distortion, the rejection and distortion increasing as a function of N.

The outputs of the various stack routines were displayed using the CalComp 663 digital plotter. Here each output trace is the result of assuming different spread velocities. The sensitivity of the beamforming filter is limited by the long wavelegths (high velocity, low frequency) and the shortness of the spread. A geophone separation of one or two kilometers would have been more appropriate, as seen in retrospect. However, the results were satisfactory, as verified by the figures.

Let us consider the stacking procedures in more detail. Consider K channels of data,

$$X_j(t) \qquad j = 1, K \qquad\qquad 2.1$$

with channel j having gain G. Let the channels be sampled at m samples per second for T seconds, giving us a K by M matrix

$$X_{i,j} \qquad i = 1, M ; \qquad j = 1, K \qquad 2.2$$

where M is the total number of samples per channel and is given by

$$M = Tm \qquad\qquad 2.3$$

and i is the time index. To bring all traces to a common
gain G, let

$$X_{i,j} = (G/G_j) X_{i,j} \qquad i = 1, M ; \quad j = 1, K \qquad 2.4$$

Let the distance of the jth geophone from the center of
the spread be D_j. Then, assuming a velocity V for the
incoming wave, the lag τ_j of channel j in samples per
second is given by

$$\tau_j = (D_j / V) m \qquad\qquad 2.5$$

The resultant of the normal stack is given by

$$R_i = (1/C) \sum_{j=1}^{K} C_j X_{i+\tau_j, j} \qquad\qquad 2.6$$

where

$$C = \sum_{j=1}^{N} C_j \qquad\qquad 2.7$$

and C_j is the weighting coefficient of the jth trace.

For the Nth root stack, the intermediate result is

$$B_{N,i} = \left(\tfrac{1}{C}\right) \sum_{j=1}^{K} C_j \, X_{i+\tau_j,j} \left| C_j \, X_{i+\tau_j,j} \right|^{\frac{1}{N+1}} \qquad 2.8$$

and the final result is

$$R_{N,i} = B_{N,i} \left| B_{N,i} \right|^{N-1} \qquad 2.9$$

This can be summed up as

$$R_{N,i} = \left[\left(\tfrac{1}{C}\right) \sum_{j=1}^{K} \left(C_j \, X_{i+\tau_j,j} \right)^{1/N} \right]^{N} \qquad 2.10$$

Note how the sign is conserved in the Nth root (equation 2.8) and the Nth power (equation 2.9) operations.

Let us consider the suppression of a spike of amplitude A at $i=I$ on channel J. Assuming all signals are the same except for phase, we have, letting $C_j=1$ for all j,

$$R_i = \left(\tfrac{1}{K}\right) \sum_{j=1}^{K} X_{i-\tau_j,j} \qquad 2.11$$

$$= \left(\tfrac{1}{K}\right) \sum_{\substack{j=1 \\ j \neq J}}^{K} S_i + S_i + A\,\delta_{i-I} \qquad 2.12$$

where $\delta_i = \begin{cases} 0 & \text{if } i \neq 0 \\ 1 & \text{if } i = 0 \end{cases}$

and S_i is the common signal with the phase difference removed. Therefore

$$R_i = S_i + (1/K) A \delta_{i-I} \qquad 2.13$$

The normal stack has reduced the amplitude of the spike by a factor of K, which is the number of traces. If the recorded signal consisted of a wanted signal and noise as represented by a Gaussian distribution of spikes, it can be shown that the signal to noise ratio is improved by a factor of $\frac{1}{2}\sqrt{K}$ (Smith, 1956).

Similarly, for the Nth root stack,

$$B_{N,i} = \left(\frac{1}{K}\right) \sum_{j=1}^{K} \left| x_{i-\tau_j, j} \right|^{1/N} \qquad 2.14$$

$$= \left(\frac{1}{K}\right) \sum_{\substack{j=1 \\ j \neq J}}^{K} S_i^{1/N} + \left(S_i + A \delta_{i-I}\right)^{1/N} \qquad 2.15$$

$$= \left(\frac{K-1}{K}\right) S_i^{1/N} + \left(\frac{1}{K}\right)\left(S_i + A\delta_{i-I}\right)^{1/N} \qquad 2.16$$

If S_I is vanishingly small as compared to A, then a first order approximation can be used, giving us

$$S_z + A \doteq A \qquad\qquad 2.17$$

Therefore

$$B_{N,i} \doteq S_i^{1/N} + (1/K) \, A^{1/N} \, \delta_{i-z} \qquad\qquad 2.18$$

and

$$R_{N,i} \doteq \left(S_i^{1/N} + (1/K) \, A^{1/N} \, \delta_{i-z} \right)^N \qquad\qquad 2.19$$

Using the same approximation

$$R_{N,i} \doteq S_i + \left(1/K \right)^N A \qquad\qquad 2.20$$

Therefore spike rejection varies as K^{-N}. However, in using the crude approximation (equation 2.17), the filter operator was changed into a linear one, whereas, in fact, the filter is highly non-linear. A much more sophisticated treatment is necessary to gain any more than the general behaviour of these filters as presented here and in figure 2.10.

2.5   Display of Results

The display is just as important as the elimination of noise, because the organization of the display plays a major role in signal interpretation. All displays

Figure 2.10   Spike attenuation using the Nth root stack

on K channels of input.

N^{th} ROOT STACK — ATTENATION OF A SPIKE

INPUT: Signal of amplitude S
        Spike of amplitude A

Channel 1 _____ S

Channel 2 _____ S

        .
        .

Channel K _____ S
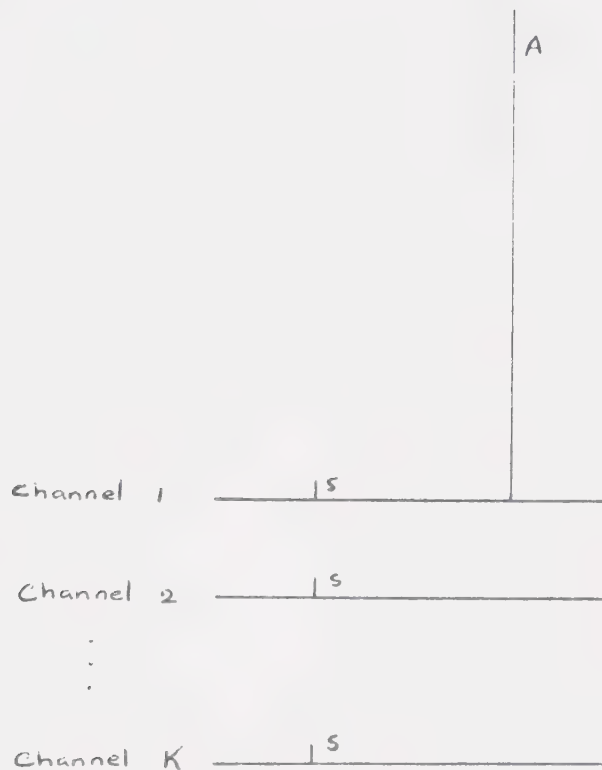
OUTPUT: Signal Amplitude $S' = S$
         Spike Amplitude $A' = AK^{-N}$

$N = 2$ $N = 8$

For: $K = 2$ _____ S _____ $A' = \frac{1}{4} A$ _____ S _____ $A' = \frac{1}{256} A$

$K = 6$ _____ S _____ $A' = \frac{1}{36} A$ _____ S _____ $A' = \frac{1}{1,679,616} A$

$K = 10$ _____ S _____ $A' = \frac{1}{100} A$ _____ S _____ $A' = \frac{1}{10^8} A$
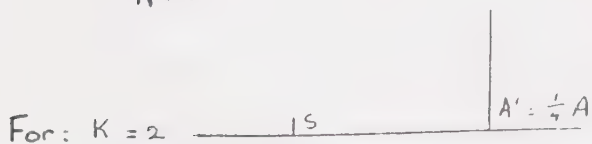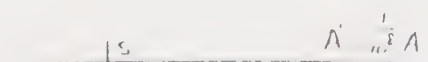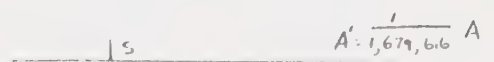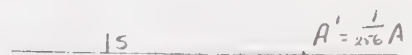
were generated on the IBM 360/67 computer and plotted
using a CalComp model 663 digital plotter. Even the
unfiltered records could be improved by organizing the
record display, so that the center axis of each trace was
displaced from the time axis a distance proportional to
the displacement of the corresponding geophone from the
center of the spread; hence the apparent spread velocity
could be read directly from the display. This proportional
factor and the gain level could be adjusted to ensure that
each trace was distinct from another and event character,
frequency, and amplitude could readily be compared on all
records. This was not necessarily the case with the field
monitor records, particularly when the recording was made
in the reverse mode; in this case the traces were
scrambled, and it required the flexibility of the CalComp
plotter to display these records in a meaningful manner.

Signal interpretation involves the identification
and timing of events, which could not be done on the
computer. An experienced observer, through a visual
display, is able to perform complex and poorly understood
(therefore unprogrammable) correlation functions to
identify these events and trace them across the profile,
provided that the signal to noise ratio is large enough.
The enhancement techniques are used to unmask the signal
from the noise, but cannot by themselves interpret the

data; hence the importance of the visual displays. They are the tools of the interpreter.

Figures 2.5 to 2.9 are characteristic of the displays of individual records. To give a picture of the entire profile, a reduced travel time composite plot (figures 2.11 and 2.12) was composed using the best recordings, which were band-pass filtered and stacked. These and similar displays of the other records were then used in analysing the data.

Figure 2.11   Composite plot of velocity filtered records.

REDUCED TRAVEL TIME COMPOSITE
NORMAL STACK 8.5 KM/SEC
FILTER 0.5 - 5.0 HZ

DISTANCE KM
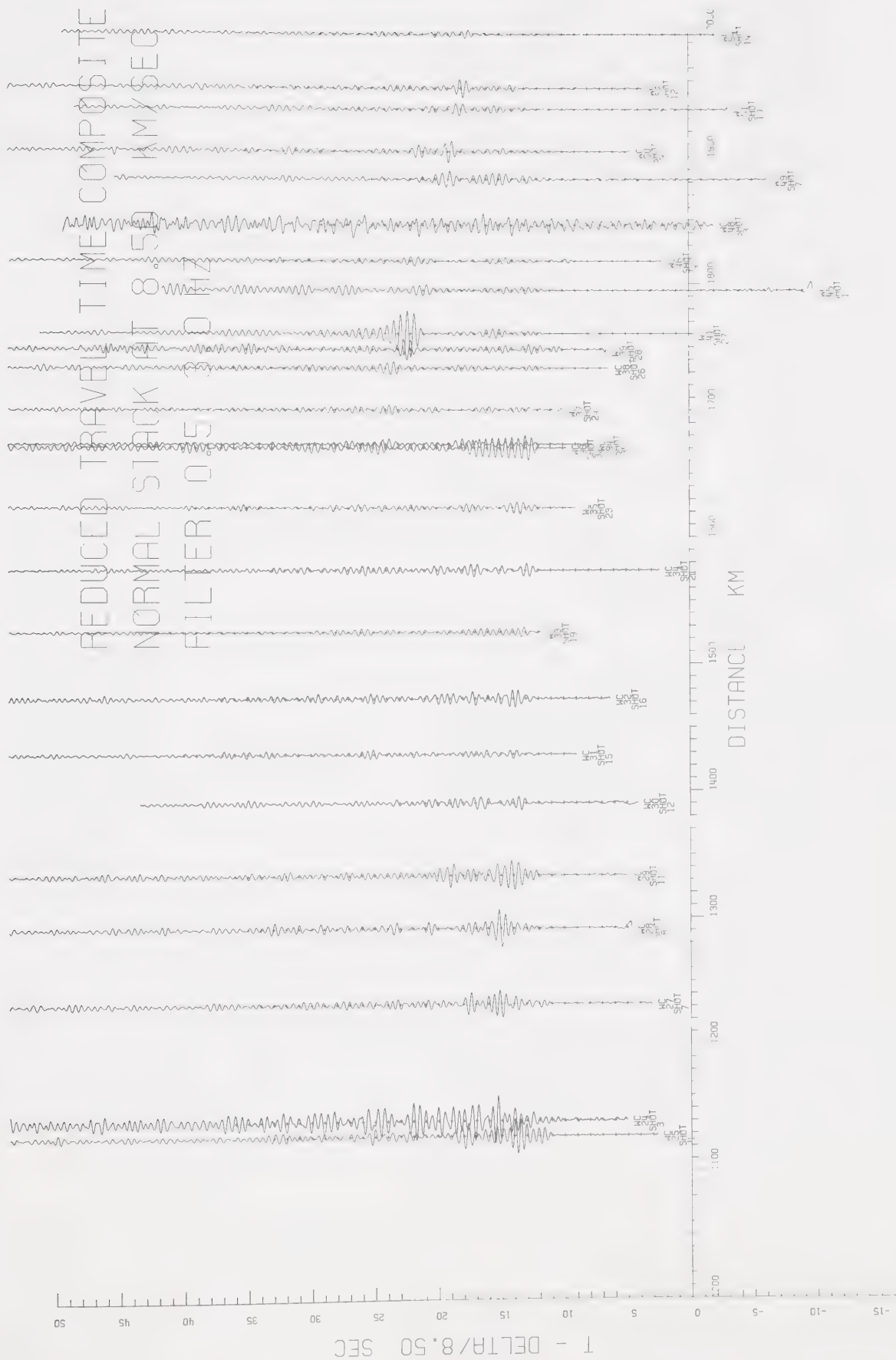
T - DELTA/8.50 SEC

Figure 2.12   Composite plot of 8th root stack filtered
records.

REDUCED TRAVEL TIME COMPOSITE
8 ROOT STACK AT 8.50 KM/SEC
FILTER 0.5 30 HZ
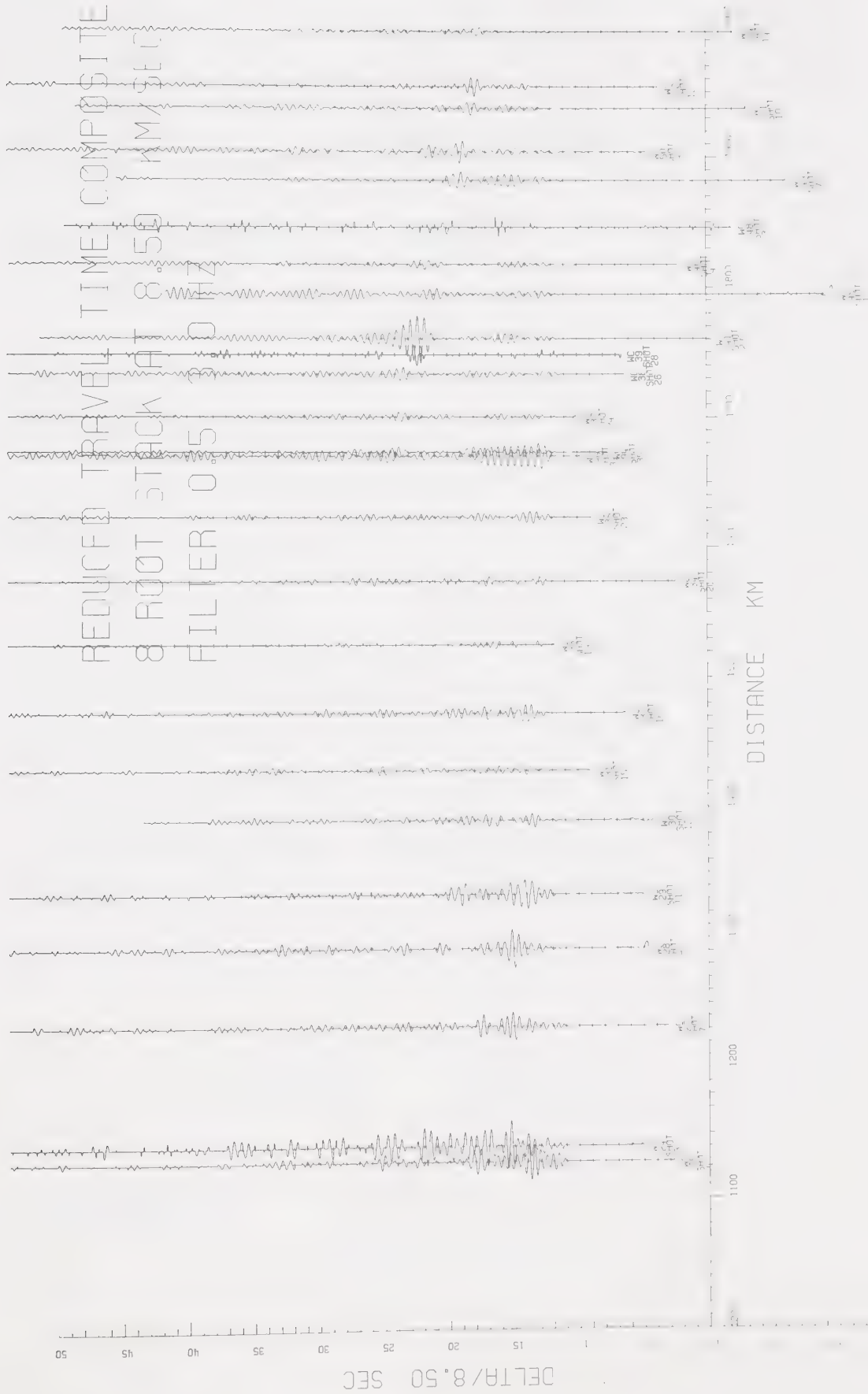
DISTANCE KM

DELTA/8.50 SEC

CHAPTER 3.

DATA INTERPRETATION

3.1   Earth Model - Ray Theory

Consider a radially symmetric earth made up
of infinitesimally thin concentric homogeneous shells
through which we shall pass a ray obeying Snell's law.
Denote the physical properties of the ith shell (between
radii r_{i-1} and r_i) with the subscript i; hence the angle
of incidence and the angle of refraction at the ith
interface can be denoted as a_i and a'_{i+1} respectively,
the angle being defined as the angle between the ray and
the normal to the interface. Further, let the intersection
of the interface and the ray be denoted as S_i (see figure
3.1). A given ray, then, can be defined by the angle of
incidence at the source, a_s, at radius r_s or depth $r_0 - r_s$.

Snell's law states

$$\frac{\sin a_i}{v_i} = \frac{\sin a'_{i+1}}{v_{i+1}}$$

3.1

From geometry

$$r_i \sin a'_{i+1} = r_{i+1} \sin a_{i+1}$$

3.2

Figure 3.1   Diagram of a ray passing through an earth
              made up of homogeneous concentric shells.

RAY PASSING THROUGH A LAYERED EARTH
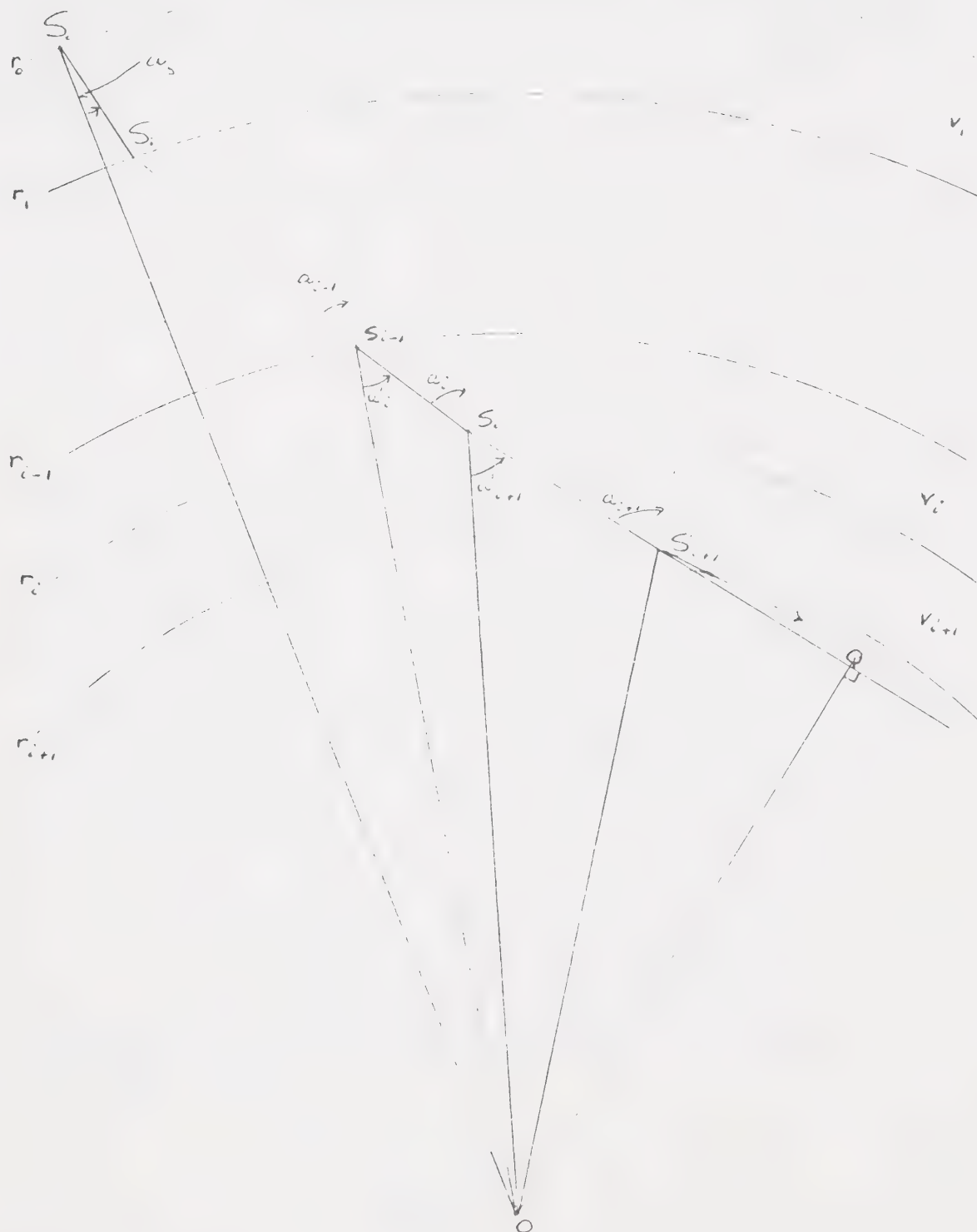
$$r_i \, \sin a'_{i+1} = QO \qquad \qquad 3.3$$

where QO is the perpendicular distance from the ray segment
in the ith shell to the center of the earth (figure 3.1).
Therefore, Snell's law becomes

$$\frac{r_i \, \sin a_i}{v_i} = \frac{r_{i+1} \, \sin a_{i+1}}{v_{i+1}} \qquad \qquad 3.4$$

$$= p \qquad \qquad 3.5$$

where p is a constant known as the ray parameter, which
is unique for each ray. Now we can drop the subscripts
and consider the velocity function v as a continuous
function of r; that is, v = v(r). Then the angle of
incidence is also a continuous function of r, (a = a(r)),
and the bottom of the ray, or the minimum radius of the
ray, r_b, is defined by

$$a(r_b) = \pi/2 \qquad \qquad 3.6$$

Hence the depth of deepest penetration may be defined
as $r_o - r_b$. Then

$$p = \frac{r \, \sin a(r)}{v(r)} \qquad \qquad 3.7$$

$$p = \frac{r_s \ \sin \ a \ (r_s)}{v \ (r_s)} \qquad 3.8$$

That is, the ray parameter is defined by the radius of
the source and the angle of incidence at the source,
and is a constant everywhere on the ray. Then by the
bottom condition stated in equation 3.6

$$p = r_b \ / \ v \ (r_b) \qquad 3.9$$

or

$$r_b = p v \ (r_b)$$

$$3.10$$

Equation 3.10 is in the form of a **trans**cendental equation
which, depending on the function $v(r)$, may or may not
be solved explicitly for r_b. In cases where it cannot
be solved explicitly, it may be solved by iteration.

Consider the ray path S originating at the
polar co-ordinates $(r_s, 0)$ and with an initial angle of
incidence a_s. An element ds of the ray path can be
expressed as

$$\vec{ds} = dr \ \vec{u_r} + r \ d\theta \ \vec{u_\theta} \qquad 3.11$$

where u_r and u_θ are unit vectors in the r, θ directions

respectvely. Two identities are immediately apparent
and are needed in the following development.

$$(ds)^2 = (dr)^2 + r^2 (d\theta)^2 \qquad\qquad 3.12$$

$$\sin a \ ds = r \ d\theta \qquad\qquad 3.13$$

whence

$$ds = r \ d\theta / \sin a \qquad\qquad 3.14$$

$$= r^2 \ d\theta / pv \qquad\qquad 3.15$$

from the definition of p given in equation 3.7. Then
substituting equation 3.15 into equation 3.12

$$(d\theta)^2 r^4 / p^2 v^2 = (dr)^2 + r^2 (d\theta)^2 \qquad\qquad 3.16$$

$$d\theta = \pm \frac{p \ dr}{r \sqrt{(r/v)^2 - p^2}} \qquad\qquad 3.17$$

Therefore the angular component of the ray path from its
source to its depth of deepest penetration is given by

$$\theta_{sb} = p \int_{r_s}^{r_b} \frac{-dr}{r \sqrt{(r/v)^2 - p^2}} \qquad\qquad 3.18$$

Solving for dθ in equation 3.13, substituting it into equation 3.12, and using equation 3.7 to eliminate a(r), we get

$$(ds)^2 = (dr)^2 + r^2 (ds \; \sin a/r)^2 \qquad \text{3.19}$$

$$= (dr)^2 + r^2 (ds \; pv/r^2)^2 \qquad \text{3.20}$$

$$= (dr)^2 + (pv/r)^2 (ds)^2 \qquad \text{3.21}$$

or

$$ds = \pm \frac{dr}{\sqrt{1 - (pv/r)^2}} \qquad \text{3.22}$$

$$= \pm \frac{r \, dr}{v \sqrt{(r/v)^2 - p^2}} \qquad \text{3.23}$$

The time dt taken to traverse the path length ds is given by

$$dt = ds/v \qquad \text{3.24}$$

or

$$dt = \pm \frac{r \, dr}{v^2 \sqrt{(r/v)^2 - p^2}} \qquad \text{3.25}$$

Therefore the time taken to traverse the ray path from the source to the depth of deepest penetration is given by

$$T_{sb} = \int_{r_s}^{r_b} \frac{-r\,dr}{v^2\sqrt{(r/v)^2 - p^2}} \qquad 3.26$$

which can be denoted in the form

$$T_{sb} = \int_{r_s}^{r_b} f_T(r)\,dr \qquad 3.27$$

and

$$\theta_{sb} = \int_{r_s}^{r_b} f_\theta(r)\,dr \qquad 3.28$$

For a complete ray path, from source to receiver,

$$T_{sr} = \int_{r_s}^{r_b} f_T(r)\,dr - \int_{r_b}^{r_r} f_T(r)\,dr \qquad 3.29$$

$$\theta_{sr} = \int_{r_s}^{r_b} f_\theta(r)\,dr - \int_{r_b}^{r_r} f_\theta(r)\,dr \qquad 3.30$$

where r_r is the radius of the receiver. When $r_s = r_r$, this simplifies to the form

$$T_{sr} = 2 \int_{r_s}^{r_b} f_T(r) \, dr \qquad 3.31$$

$$\theta_{sb} = 2 \int_{r_s}^{r_b} f_\theta(r) \, dr \qquad 3.32$$

The equations of θ and T in the forms of equations 3.29 and 3.30 are the most useful, since different velocity functions may be used at different angular displacements from the source. That is, $v(r)$ can be varied as a function of θ and becomes $v(r,\theta)$. However, unless the ray parameter p is also varied, it is still assumed that the shells are radially symmetrical, but that the radius of the shell can be made to vary with θ for individual ray paths. In other words, the "dip" of the shell has not been taken into account unless the ray parameter p is suitably varied. Hence the usefulness of equations 3.29 and 3.30 is limited to varying the elevation and crustal structure under the source and the receiver, still assuming beds parallel to the surface of the earth. This can be accomplished by using the source velocity function in the down integral (r_s to r_b) and using a suitable velocity function in the up integral as based on the expected angular displacement from the source to the receiver end of the ray, as estimated by $2\theta_{sb}$ from equation 3.32. It should be possible to consider an asymmetric earth by varying the ray parameter p as

a function of the dip of the velocity shells as represented
by sherical shells displaced from the center of the
earth.

3.2   Solutions of the Ray Path Integrals

Note that both θ and T are of the form

$$X = \int_A^B f(x) \; dx \qquad\qquad 3.33$$

There are several ways of evaluating this integral.
The obvious method is to evaluate

$$F(x) = \int f(x) \; dx \qquad\qquad 3.34$$

and then

$$X = F(B) - F(A) \qquad\qquad 3.35$$

However, f(x), or rather $f_\theta(r)$ and $f_T(r)$, are not well
known functions, as v(r) is assumed at discrete values
of r, and some form of extrapolation is used between
these points. Therefore v(r) is, at best, a continuous
function with discontinuous high order derivatives. Also,
integral solutions exist for only a limited number of

velocity functions, and these may not necessarily be physically realistic.

Let us consider a finite number of concentric homogeneous shells. When $v(r) = v_i$, a constant, for $r_{i-1} \leqq r < r_i$, then within this interval equation 3.18 becomes

$$\theta_{i,i-1} = p \int_{r_i}^{r_{i-1}} \frac{-dr}{r\sqrt{(r/v_i)^2 - p^2}} \qquad 3.36$$

$$= p\,v_i \int_{r_i}^{r_{i-1}} \frac{-dr}{r\sqrt{r^2 - p^2 v_i^2}} \qquad 3.37$$

$$= \cos^{-1}\left(\frac{p\,v_i}{r_i}\right) - \cos^{-1}\left(\frac{p\,v_i}{r_{i-1}}\right) \qquad 3.38$$

Let I be an integer such that $r_{I-1} < r_b \leqq r_I$ and note that $r_b \geqq pv_I$. The equality holds for the refraction and the inequality for the reflection of the ray from the Ith interface. Hence, for refraction,

$$\cos^{-1}\left(p\,v_I/r_b\right) = \cos^{-1}(1) \qquad 3.39$$

$$= 0 \qquad 3.40$$

and must be set to zero, since a small round off error could result in an argument greater than 1 and hence an

imaginary angle. Therefore

$$\theta_{sb} = \sum_{i=1}^{I-1} \cos^{-1}\left(\frac{p v_i}{r_i}\right) - \cos^{-1}\left(\frac{p v_i}{r_{i-1}}\right)$$

$$+ \cos^{-1}\left(\frac{p v_I}{r_{I-1}}\right) - \cos^{-1}\left(\frac{p v_I}{r_b}\right) \qquad 3.41$$

and

$$\theta_{br} = -\cos^{-1}\left(\frac{p v'_J}{r_b}\right) + \cos^{-1}\left(\frac{p v'_J}{r_{J-1}}\right)$$

$$+ \sum_{j=J-1}^{1} \cos^{-1}\left(\frac{p v'_j}{r_j}\right) - \cos^{-1}\left(\frac{p v'_j}{r_{j-1}}\right) \qquad 3.42$$

where v'_j may be a new velocity step function and J is
an integer such that $r_{J-1} < r'_b \le r_J$ and $r_b \ge p v'_J$.
If $r'_b \ne r_b$, then the bottom of the ray is in the
region where $v = v(r,\theta)$ and the ray parameter must be
adjusted from the old value p to p' such that either

    i) in the case of reflection from the Ith interface

$$\sin^{-1}\left(p v_I / r_b\right) = \sin^{-1}\left(p' v'_J / r_b\right) \qquad 3.43$$

or

$$p' = p v_I / v'_J \qquad 3.44$$

which is derived from the requirement that the angle of

incidence be equal to the angle of reflection, still assuming concentric shells, or

    ii) in the case of refraction

$$p' = r_b / \sigma'_J \tag{3.45}$$

$$= p\sigma_I / \sigma'_J \tag{3.46}$$

which follows from the requirement that the ray be travelling tangential to the curve of $r = r_b$. This can be considered as a special case of reflection where the angle of incidence is $\pi/2$.

    Let us consider the solution to the travel time.

$$T_{i,i-1} = -\int_{r_i}^{r_{i-1}} \frac{r\,dr}{\sigma_i \sqrt{r^2 - p^2 \sigma_i^2}} \tag{3.47}$$

$$= -\frac{1}{\sigma_i} \int_{r_i}^{r_{i-1}} \frac{r\,dr}{\sqrt{r^2 - p^2 \sigma_i^2}} \tag{3.48}$$

$$= \frac{\sqrt{r_i^2 - p^2 \sigma_i^2}}{\sigma_i} - \frac{\sqrt{r_{i-1}^2 - p^2 \sigma_i^2}}{\sigma_i} \tag{3.49}$$

and

$$T_{sb} = \sum_{i=1}^{I-1} \frac{\sqrt{r_i^2 - p^2 v_i^2} - \sqrt{r_{i-1}^2 - p^2 v_i^2}}{v_i}$$

$$+ \frac{\sqrt{r_{I-1}^2 - p^2 v_I^2} - \sqrt{r_b^2 - p^2 v_I^2}}{v_I} \qquad 3.50$$

and

$$T_{br} = \frac{-\sqrt{r_b^2 - p'^2 v_J'^2} + \sqrt{r_{J-1}^2 - p'^2 v_J'^2}}{v_J'}$$

$$+ \sum_{j=J-1}^{1} \frac{\sqrt{r_j^2 - p'^2 v_j'^2} - \sqrt{r_{j-1}^2 - p'^2 v_j'^2}}{v_j'} \qquad 3.51$$

Again note that in the case of refraction $r_b = pv_I$ and the calculation of the bottom term must be set to zero, since a small error could result in an imaginary square root.

A solution for θ exists when a continuous velocity distribution of the form

$$v_i(r) = \alpha_i + \beta_i r \qquad 3.52$$

is assumed between the shell radii r_{i-1} and r_i. Then

$$\theta_{i,i-1} = -p \int_{r_i}^{r_{i-1}} \frac{v_i \, dr}{r \sqrt{r^2 - p^2 v_i^2}} \qquad 3.53$$

$$\theta_{i,i-1} = -\rho \int_{r_i}^{r_{i-1}} \frac{(\alpha_i/r + \beta_i)\, dr}{\sqrt{(1 - \rho^2 \beta_i^2)\, r^2 - 2\rho^2 \alpha_i \beta_i r - \alpha_i^2 \rho^2}} \qquad 3.54$$

Let

$$a_i = -\rho^2 \alpha_i^2 \qquad\qquad 3.55$$

$$b_i = -2\rho^2 \alpha_i \beta_i \qquad\qquad 3.56$$

$$c_i = 1 - \rho^2 \beta_i^2 \qquad\qquad 3.57$$

and

$$R_i(r) = a_i + b_i r + c_i r^2 \qquad\qquad 3.58$$

Then

$$b_i^2 - 4a_i c_i = 4\rho^2 \alpha_i^2 \qquad\qquad 3.59$$

Whence

$$\theta_{i,i-1} = \rho \alpha_i \int_{r_i}^{r_{i-1}} \frac{(r)\, dr}{\sqrt{R_i}\, (r)}$$

$$+ \rho \beta_i \int_{r_i}^{r_{i-1}} \frac{(r)\, dr}{\sqrt{R_i}} \qquad\qquad 3.60$$

Let $\theta'_{i, i-1}$, $\theta''_{i, i-1}$ represent the first and second terms respectively. Since $a < 0$ for all p, α_i, then

$$\theta'_{i, i-1} = \sin^{-1}\left(\frac{-p\beta_i \, r_{i-1} + p\alpha_i}{r_{i-1}}\right)$$

$$- \sin^{-1}\left(\frac{-p\beta_i \, r_i + p\alpha_i}{r_i}\right) \qquad 3.61$$

For $c < 0$,

$$\theta''_{i, i-1} = \frac{p\beta_i}{\sqrt{p^2\beta_i^2 - 1}}\left[\sin^{-1}\left(\frac{(p^2\beta_i^2 - 1)\, r_{i-1} + p^2\alpha_i\beta_i}{p\alpha_i}\right)\right.$$

$$\left. - \sin^{-1}\left(\frac{(p^2\beta_i^2 - 1)\, r_i + p^2\alpha_i\beta_i}{p\alpha_i}\right)\right] \qquad 3.62$$

and for $c > 0$,

$$\theta''_{i, i-1} = \frac{p\beta_i}{\sqrt{c_i}} \, \ln\left[\sqrt{R_i}(r_{i-1}) + r_{i-1}\sqrt{c_i} + \frac{b_i}{2\sqrt{c_i}}\right]$$

$$- \frac{p\beta_i}{\sqrt{c_i}} \, \ln\left[r_i\sqrt{R_i} + r_i\sqrt{c_i} + \frac{b_i}{2\sqrt{c_i}}\right] \qquad 3.63$$

Therefore

$$\theta_{sb} = \sum_{i=1}^{I-1} \theta'_{i-1, i} + \theta''_{i-1, i} + \theta'_{I-1, I^*} + \theta''_{I-1, I^*} \qquad 3.64$$

where we define $r_{I^*} = r_b$ and $r_{i-1} < r_b \leq r_I$.
Similarly, on changing the velocity function, we obtain

$$\theta_{br} = \theta'_{J-1, J\bullet} + \theta''_{J-1, J\bullet} + \sum_{j=J-1}^{1} \theta'_{j-1,j} + \theta''_{j-1,j} \qquad 3.65$$

where again the p parameter is reset to

$$p' = r_b \Big/ v'(r_b) \qquad\qquad 3.66$$

The equivalent integral for time using $v = \alpha + \beta r$ cannot be readily integrated, and no solution has been derived here.

As expressed in equations 3.18 and 3.26, both θ and T are of the form $X = \int_A^B f(x)\,dx$. When the function $v(r)$ is not well known and various methods of extrapolation between radii of assumed velocities are used, then a center point integration formula of the form

$$X = \sum_{i=0}^{N} f(x_{ci}) \, \Delta x \qquad\qquad 3.67$$

$$= (B - A) \sum_{i=0}^{N} f(x_{ci}) \qquad\qquad 3.68$$

may be used, where N is an arbitrary positive integer and

$$\Delta x = (B - A) / N \qquad\qquad 3.69$$

$$X_{ci} = \tfrac{1}{2}(x_i + x_{i+1}) \qquad\qquad 3.70$$

$$x_i = A + i \Delta x \qquad\qquad 3.71$$

Using this method, the integration of $f(x)$ is equated to the area under the curve $f(x)$, and this is approximated by the sum of the areas of several rectangles of height $f(x_{ci})$; $i = 0,N$ and of width Δx. Both positive and negative errors occur which tend to cancel each other (see figure 3.2), and which are further reduced by decreasing Δx. The accuracy is then limited by the accumulated round off errors, which increase as N is increased and hence as Δx is decreased.

With a knowledge of the behaviour of the function $f(x)$, it is possible to vary the size of Δx for different ranges of x. In our case, we are integrating down to a pole, the pole being the radius at which the ray bottoms. Far from the pole relatively large steps may be taken with small sacrifice to the accuracy of the approximation and with a large decrease in the accumulated round off error. As the pole is approached, the curvature of the function increases. Hence, smaller steps are used, and the error of the approximation is reduced. Also, the round off error is small, because each of the terms representing the area under the curve is of small width but of large height, therefore of considerable area. Another range of x exists, where the function is of considerable height and increasing rapidly. This occurs

Figure 3.2   Center point integration formula approximation
to the area under the curve.
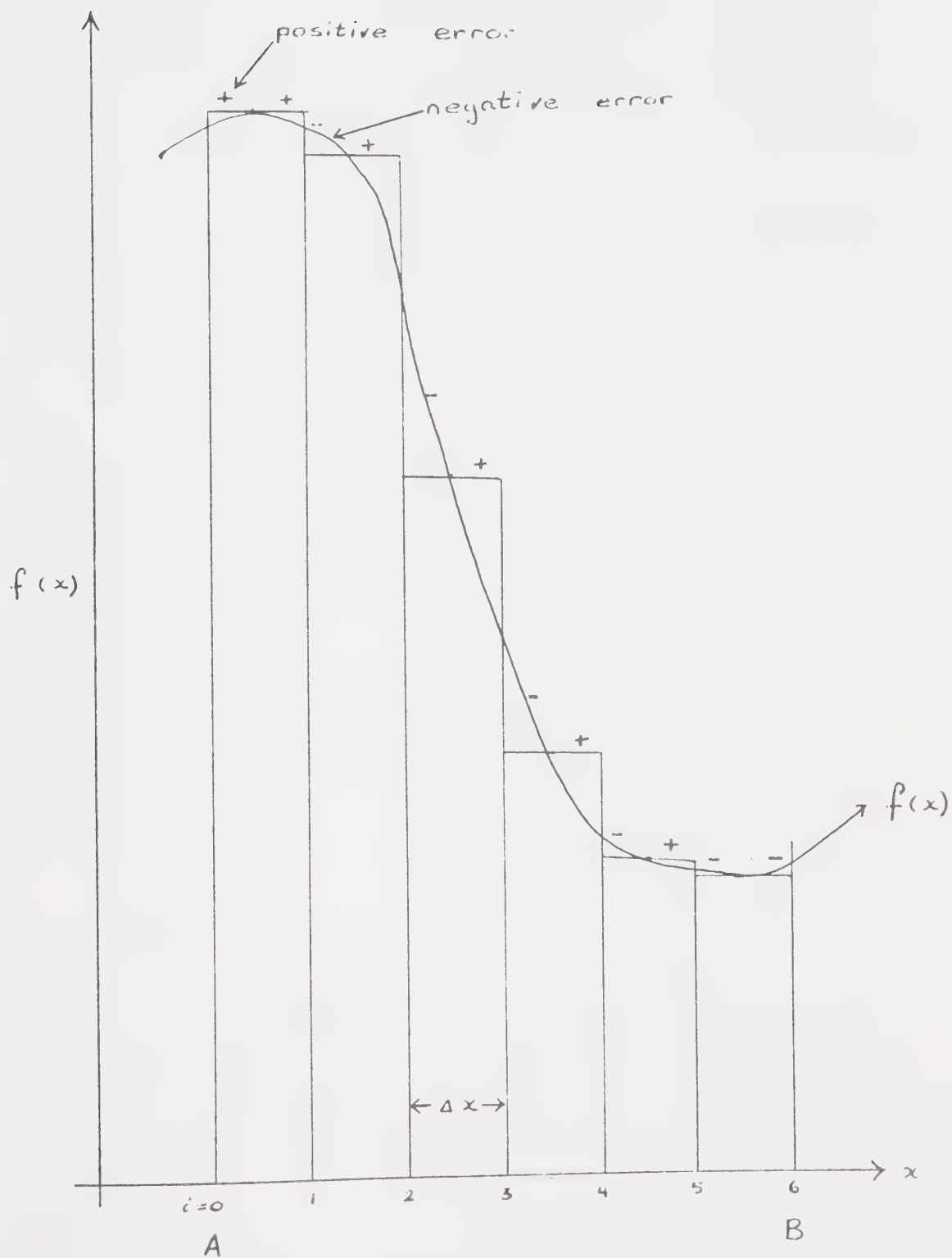
CENTER POINT INTEGRATION APPROXIMATION

when the pole exists just below a low velocity shell, in
which case there is a region immediately above this shell
where finer increments in the integration are required.
Thus by suitably varying Δx, it is possible to minimize
both the error of the approximation and of the round off.

Fortran programs were written (see appendix)
using both the thin shell constant velocity and the
center point integration formaulae. To check the accuracy
of the programs, they were used to reproduce the travel
times of Jeffreys and Bullen(1958); Lewis and Meyer
(1967); Johnson (1967); and Herrin, Tucker, Taggart,
Gordon, and Lobdell (1968). Furthermore, the two
methods closely agreed with one another in each of the
above checks. In addition, the center point integration
formula agreed closely with the exact solution of the
constant velocity formula in the case of a homogeneous
earth.

3.3   Ray Flux Calculations

In conjunction with the travel time curves,
ray paths and amplitude predictions were made. The ray
paths were calculated by evaluating r, θ in each step of
the integral and the results were plotted using the
CalComp digital plotter. The energy density was calculated
from the ray flux, which was shown by Bullen (1963) to be

$$E(\theta) = \frac{I \sin a_s}{r_r^2 \sin\theta \sin a_r} \left| \frac{da_s}{d\theta} \right| \qquad 3.72$$

where I is a constant of proportionality denoting the energy per solid angle emitted by the source, a_s is the angle of incidence at the source, a_r is the angle of emergence at the receiver, r_r is the radius of the receiver, θ is the angular distance from source to receiver, and da_s, $d\theta$ are increments in a_s, θ respectively. To derive this equation we consider all the rays leaving the source between the angles of incidence a_s and $a_s + da_s$ (see figure 3.3). The energy radiated in this ray bundle is $2\pi I \sin a_s |da_s|$, the modulus of da_s being used to insure that this term is positive. This ray bundle arrives at the receiver radius r_r between the distances θ and $\theta + d\theta$, which sweeps out an area on a sphere of radius r_r given by $2\pi r_r^2 \sin\theta |d\theta|$. Again, the modulus of $d\theta$ is used, as $d\theta$ may be negative, whereas the area is always positive. Then the cross-sectional area of the ray bundle is given by $2\pi r_r^2 \sin\theta \cos a_r |d\theta|$ and the energy density at the angular distance θ is

$$E(\theta) = \frac{2\pi I \sin a_s |da_s|}{2\pi r_r^2 \sin\theta \cos a_r |d\theta|} \qquad 3.73$$

$$= \frac{I \sin a_s}{r_r^2 \sin\theta \cos a_r} \left| \frac{da_s}{d\theta} \right| \qquad 3.74$$

Figure 3.3   Diagram of two rays passing through the
earth, showing the change in the angular
displacement of the emerging ray for a
small change in the angle of incidence.

DIVERGENCE OF A RAY BUNDLE
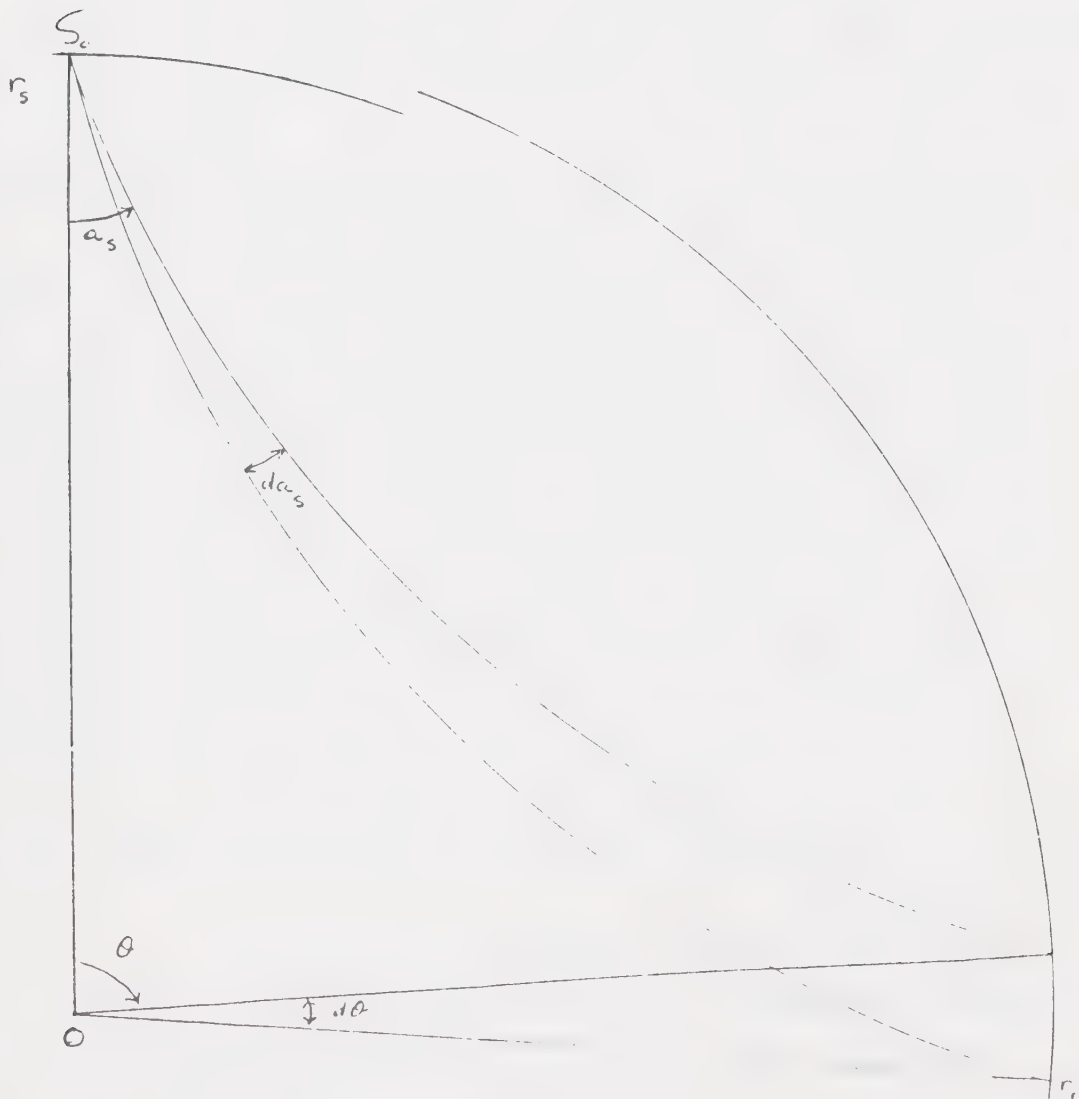
Finally, the square of the amplitude is proportional to
the energy density, whereby

$$A(\theta) = I_1 \sqrt{E(\theta)} \qquad\qquad 3.75$$

$$= I_1 \sqrt{\frac{I \sin a_s}{r_r^2 \sin \theta \cos a_r} \left| \frac{d a_s}{d \theta} \right|} \qquad 3.76$$

$$= \frac{I_2}{r_r} \sqrt{\frac{\sin a_s}{\sin \theta \cos a_r} \left| \frac{d a_s}{d \theta} \right|} \qquad 3.77$$

Using angles of incidence a_s, $a_s + d a_s$, and $a_s - d a_s$, rays
were passed, which arrived at the receiver radius with
an angular distance of θ_r, θ_{r+}, and θ_{r-} respectively.
Hence $d\theta$ could be estimated using the first difference
method on either side of a_s and smoothed by taking the
average of these terms. Thus all the arguments of
equation 3.77 are known and $A(\theta)$, along with the travel
time curve, can be calculated for any given velocity
distribution.

3.4   Interpretation of the Travel Time Curve

The various earth models were derived by
varying the velocity function until a good fit was made
between the theoretical travel time curves and the
events picked from the records. Model U of A #1 was fit
to the first arrivals and to a prominent secondary

arrival as reported in the Technical Letter #6 (Warren et al., 1967). The ray and travel time diagrams are shown in figures 3.4 and 3.5, the latter also showing the Lewis and Meyer (1967) model for comparison. The Lewis and Meyer profile was also a part of Project Early Rise and was run along the 49th parallel (see figure 1.1), which is close to the Western Canada profile. Therefore, the two profiles should show similar results, as in fact they do, as illustrated in the diagram.

When a full analysis of the secondary arrivals had been made, it became evident that a simple model such as a modification of U of A #1 could not explain all of the arrivals. In order to explain the shadow zone and cusping in the travel time curve, one needs velocity reversals and large velocity gradients. Model U of A #3 produced the best fit to the observed data (figures 3.4 and 3.6). This model is markedly differnt from U of A #1. By removing the sharp corners of the velocity function of U of A #3, a modification, U of A #3a, resulted. As seen in figure 3.6, there was no significant difference in the resulting travel time curves of these two models. When the ray criterion of explaining events was relaxed, and the possibility of diffraction was used to explain the extreme ends of the cusps, then U of A #2 produced the best fit to all parts of the travel time curve

Figure 3.4   Ray diagram for body waves passing through
earth models U of A #1 and U of A #3.

SEISMIC RAYS
VELOCITY MODEL U OF A # 1

SEISMIC RAYS
VELOCITY MODEL U OF A # 1
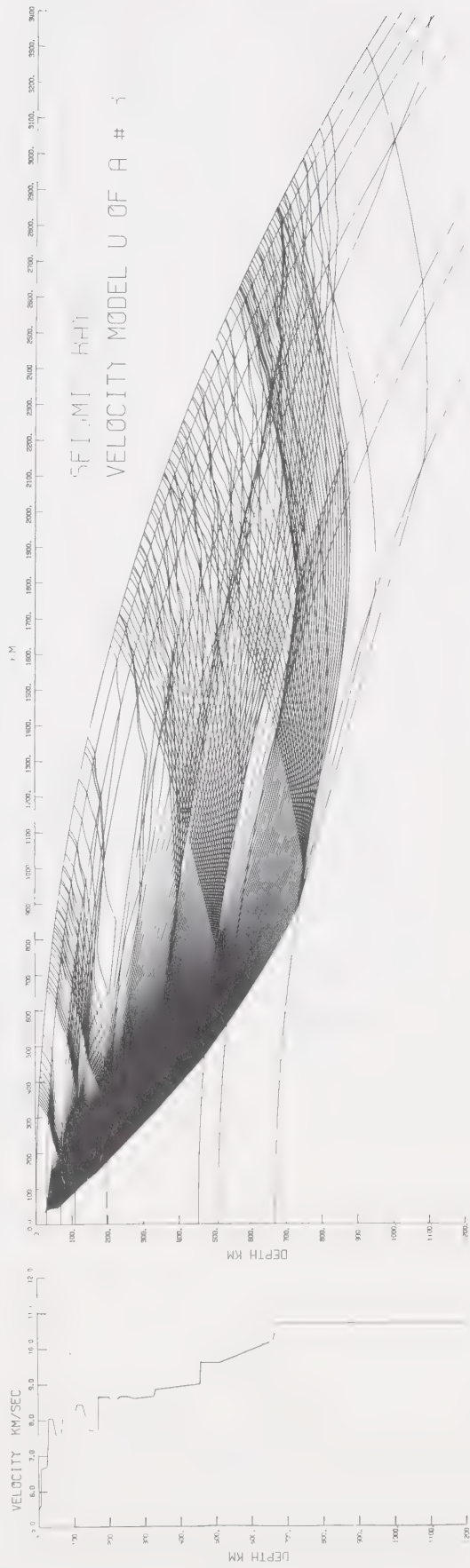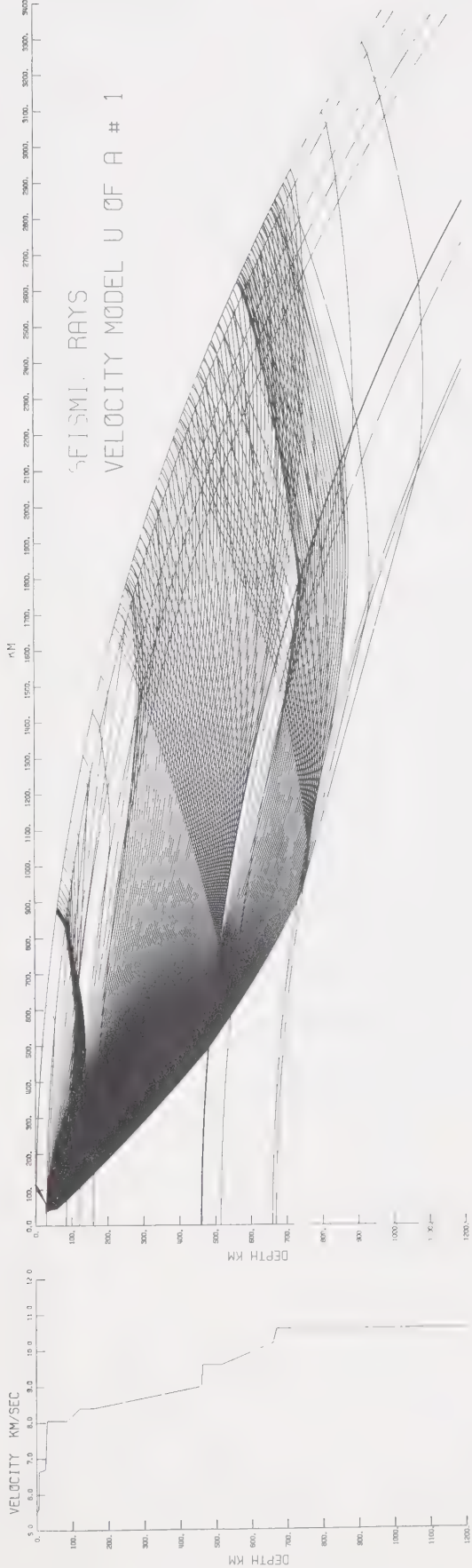
Figure 3.5   Velocity and travel time curves of the
            Lewis and Meyer model and the University
            of Alberta model U of A #1. Also shown are
            the observations of the first events and a
            prominent secondary event of the Western
            Canada profile.
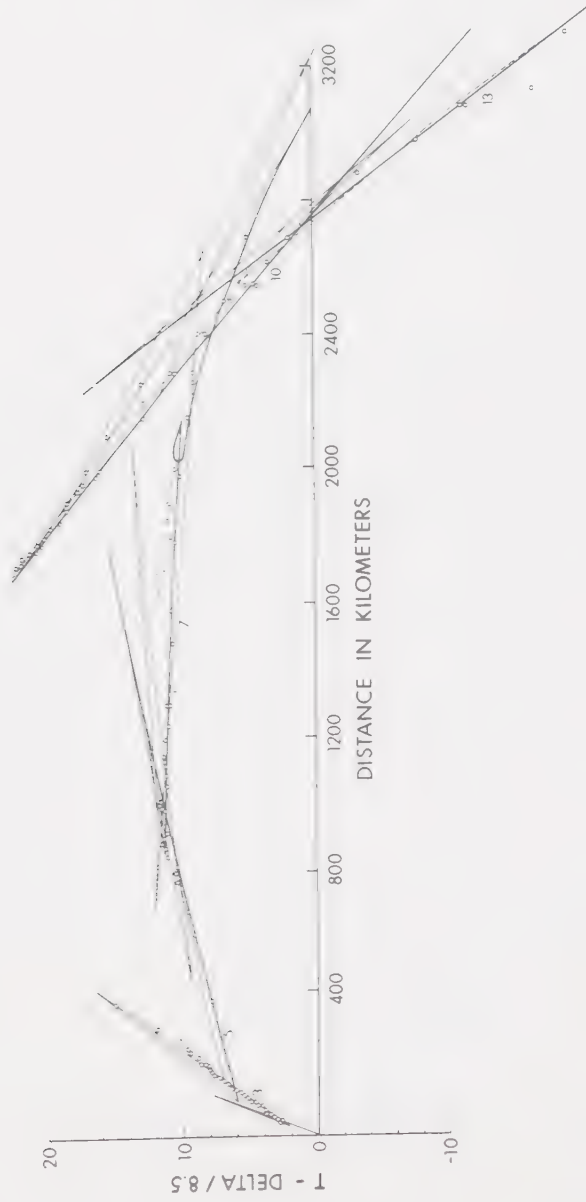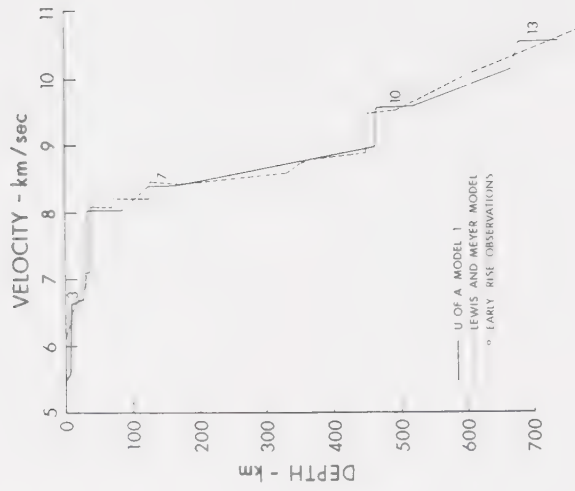
Figure 3.6   Velocity and travel time curves of the
              models U of A #2, U of A #3, and U of A #3a
              along with the Western Canada profile
   observations.
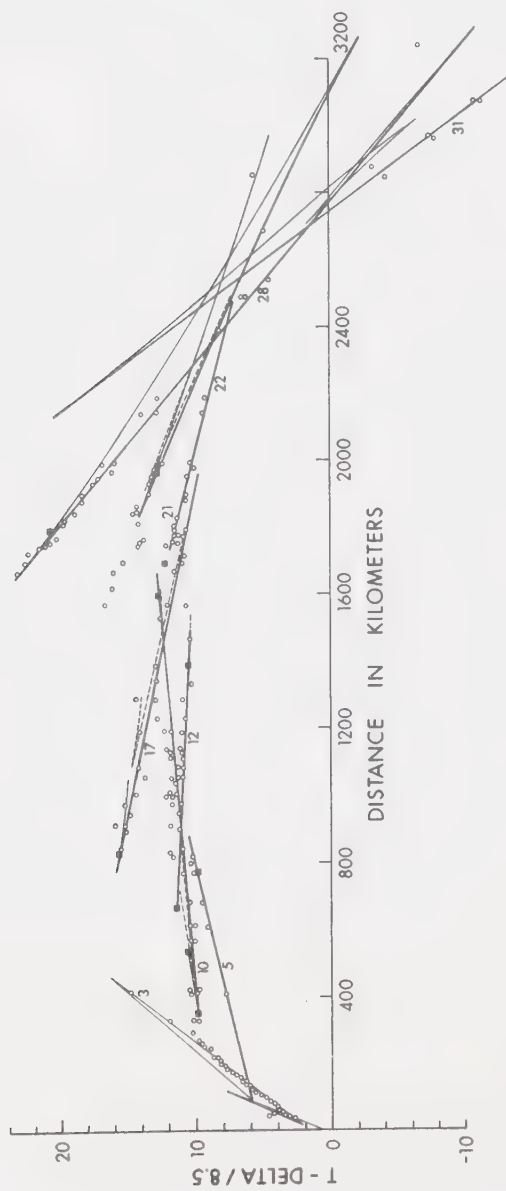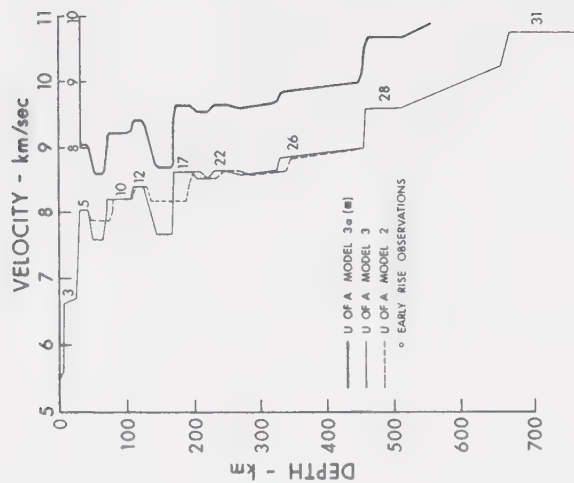
except the tips of the cusps, as shown by figures 3.6 and 3.7. The selection of a unique model to fit the data and possibly the determination of the shape of the transition zones lies in the matching the predicted amplitudes to the observed ones, as well as maintaining the fit to the observed travel time curve. This work is presently in progress.

Figure 3.8 is presented to summarize this work. It shows U of A #3, which is a model fitting a continental profile, and compares it to a model worked out by Johnson(1967), which is based on an oceanic profile. Also, a model based on a summary of the United States continental data (Herrin et al., 1968) is shown for further comparison. The rough features of our model and that of Johnson are seen to coincide, with ours showing a delay of about 2 seconds. The Herrin model lies slightly above model U of A #3 and has none of the detailed cusping of our model. Further work in this area should clarify the details of the upper mantle, but the gross features must be as represented here. In particular, a reversal of this profile should clarify the true velocities of the various zones.

Figure 3.7   Ray diagram for body waves passing through
the earth model U of A #2.

MODEL U OF H # 2
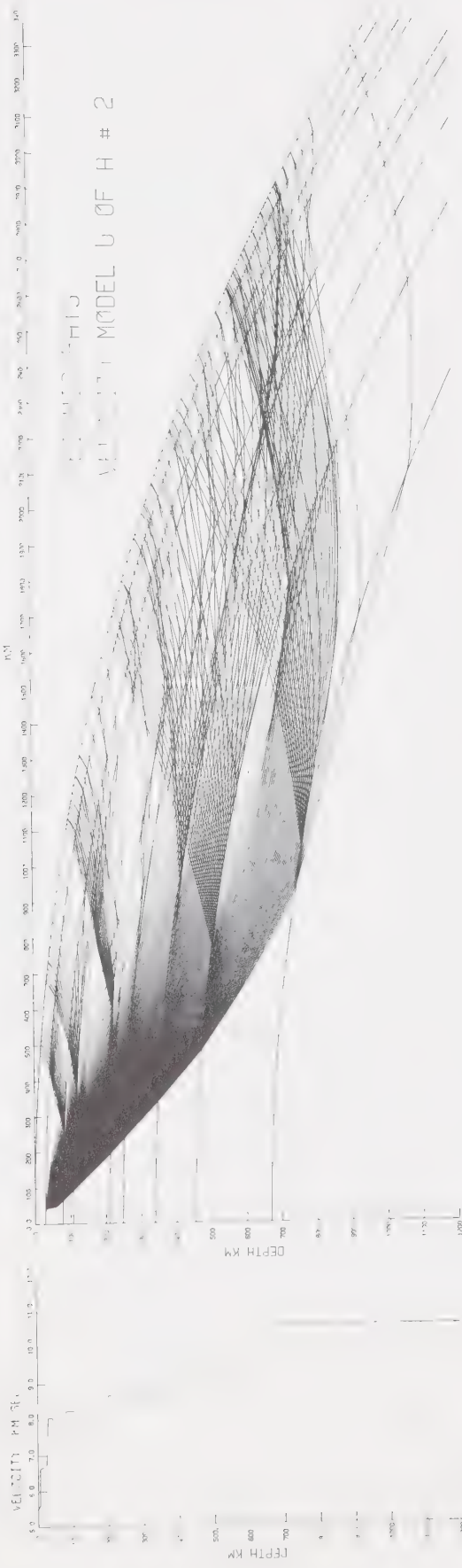
Figure 3.8   Velocity and travel time curves of the
             final model produced by the University of
             Alberta (U of A #3) along with those
             resulting from the oceanic model produced
             by Johnson and the average crustal model
             produced by Herrin, Tucker, Taggart, Gordon,
             and Lobdell.
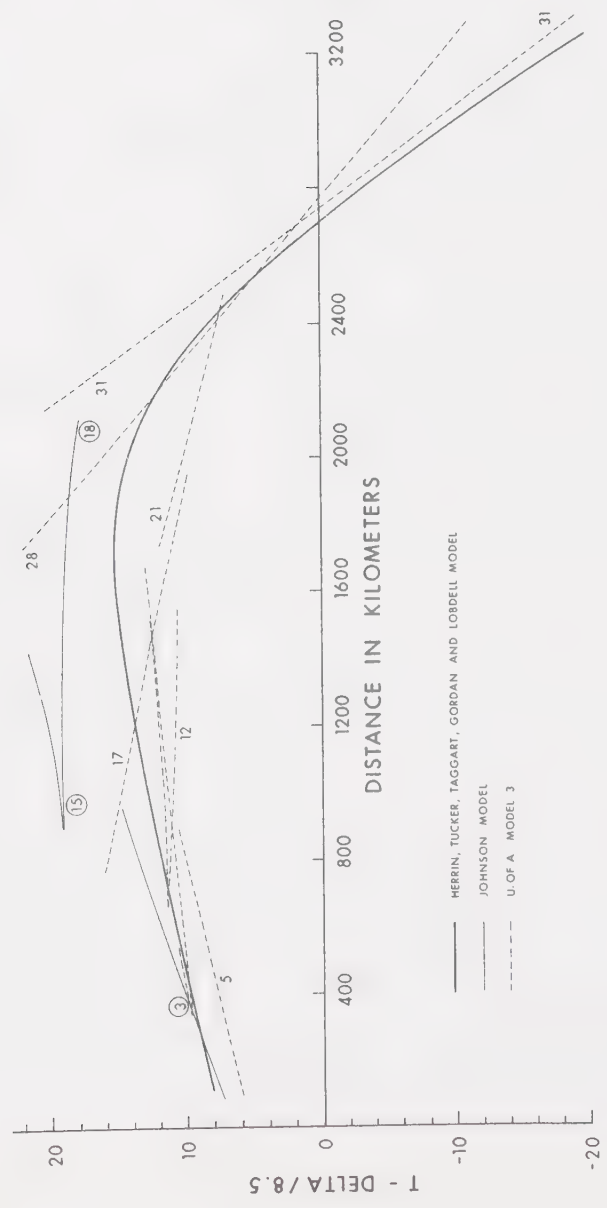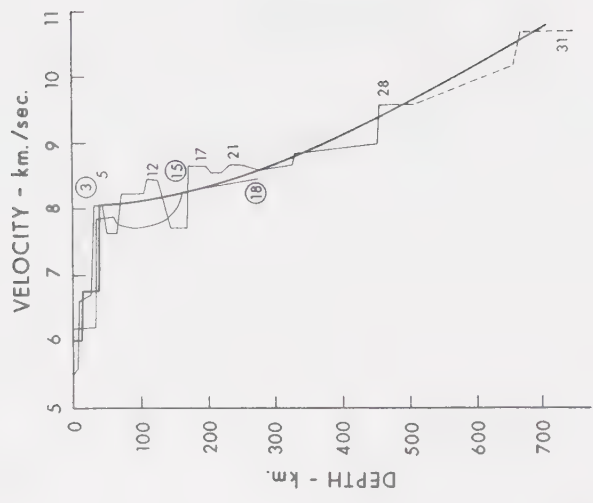
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       Gordon, and John L. Lobdell. "Estimation of Surface
       Focus P Travel Times", Bulletin of the Seismological
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       in the Upper Mantle", Journal of Geophysical
       Research, 72 #24 (December 1967), 6309-6325.

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       Investigation of the Upper Mantle to the West
       of Lake Superior", Bulletin of the Seismological
       Society of America, 58 #2 (April 1968), 565-596.

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       1968), 533-534.

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       Theory", Geophysics, XXI #2 (April 1956), 337-360.

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    Srinivasreddy Rauula, and D. J. Stuart. "Technical
    Letter #6: Project Early Rise Traveltimes and
    Amplitudes", USGS, Menlo Park, 1967.

APPENDIX.

FORTRAN IV PROGRAM FOR THE TIME AND DISTANCE INTEGRALS

The following Fortran IV program was used to calculate the travel time curve for a given earth model. It integrates from R_1 (which is equivalent to r_s in this paper) to a calculated r_b using velocity model $v(r,0)$, and then has the option of resetting the velocity structure for the up integral by evaluating the $v(r,\theta)$, where θ is twice θ_{sb}.

Two integral formulae were used; and hence the listing of two down integral and two up integral subroutines. One is the center point integration formula and the other is the thin homogeneous shell. A third up integral subroutine, which simply doubles the value of θ_{sb} and T_{sb}, is also listed. This is valid only when $v = v(r)$ and $r_s = r_b$. The appropriate subroutines must be selected by the user.

```
      REAL*4 RDEL(50,20),VDEL(50,20),DELMOD(50),RMAN(81),VMAN(81)
      REAL*8 R(101),V(101)
      REAL*8 RO(101),VO(101)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL,TIMEC
  100 FORMAT(5F10.0,3I10)
  103 FORMAT('0ANGLE OF INCIDENCE = ',F10.5)
 1040FORMAT('1RADIUS OF THE EARTH =',F10.3,'KM'/'0START INTEGRATION AT'
     1,F10.3,'KM, STOP AT',F10.3,'KM. RADIUS OF PENETRATION LIMITED TO',
     2F10.3,'KM'/ 1/DR STARTS AT',I4,' STOPS AT',I8,' & ITEST =',I8)
 1050 FORMAT('0RAY FLUX ON EACH SIDE OF THE RAY IS  ',2(D12.6,
     1'    = LOG  (', F10.5,') '),'  AV FLUX =',F10.5)
  106 FORMAT('1')
  107 FORMAT('1RAY PATH COORDINATES DELETED')
  108 FORMAT(2F10.0,I5,54X,I1)
 1100FORMAT('-',20X,'RAY STARTS',F8.2,' DEG ',5X,'RAY ENDS',F8.2,
     1' DEG ',5X,'INTERVAL NO',I5)
      J$=50
      I$=20
      K$=81
      L$=101
      RADDEG=3.1415926536D0/180.0D0
C
C     DATA CARD 1 F10.0 NRAY
C     NRAY IS THE NUMBER OF RAYS PASSED PER SHELL FOR TRAVEL TIME CURVE
C     NRAY = I IF UNSPECIFIED (BLANK FIELD)
C
      READ(5,100)XOUT
      NRAY=IFIX(XOUT)
      IF(NRAY.LT.2)NRAY=2
C
C     DATA CARD 2 F10.0 XOUT
C     IF XOUT NOT SPECIFIED, THE RAY PATH CO-ORDINATES ARE DELETED
C     IF XOUT NE 0.0 THE RAY PATH CO-ORDINATES ARE CALCULATED
C
      READ(5,100)XOUT
      IF(IFIX(XOUT).EQ.0)WRITE(6,107)
C
C     DATA CARD 3 F10.0 RE
C     RE IS THE RADIUS OF THE SPHERE
C     RE = 6371.0 (KM) IF UNSPECIFIED
C
      READ(5,100)RE
      IF(RE.EQ.0.0D0)RE=6371.0D0
    2 CONTINUE
C
C     DATA CARDS SET 1  TITLE AND DATA OF VELOCITY STRUCTURE
C     SEE SUBROUTINE DATA
C
      CALL DATA(RDEL,VDEL,DELMOD,J$,I$,RMAN,VMAN,K$,R,V,L$)
    1 CONTINUE
C
C     DATA CARD SET 1 + 4 5F10.0,3I10 R1,R2,R3,VRED,TINC,IDR1,IDR2,ITEST
```

```
C      R1 IS RADIUS (KM) OF SOURCE
C      RE IS RADIUS (KM) OF SPHERE
C      R1 = RE IF UNSPECIFIED
C      R3 IS RADIUS (KM) OF RECEIVER
C      R3 = RE IF UNSPECIFIED
C      R2 IS MINIMUM RADIUS (KM) RAY MAY PENETRATE
C      VRED IS REDUCTION VELOCITY (KM/SEC) FOR REDUCED TRAVEL TIME
C      VRED = 8.5 KM/SEC IF UNSPECIFIED
C      IDR1 IS IDR AT START OF INTEGRAL
C      WHERE DR = 1/IDR , IS THE INCREMENT IN THE INTEGRAL
C      IDR1 = 2 IF UNSPECIFIED
C      IDR2 IS IDR LIMIT FOR END OF INTEGRAL
C      IDR2 = 128 IF UNSPECIFIED
C      ITEST IS NUMBER OF INCREMENTS TO BOTTOM OF RAY (REND) BEFORE
C      IDR = IDR*2
C      ITEST = 100 IF UNSPECIFIED
C
       READ(5,100)R1,R2,R3,VRED,TINC,IDR1,IDR2,ITEST
       IF(R1.EQ.0.0D0)R1=RE
       IF(R3.EQ.0.0D0)R3=RE
       IF(VRED.EQ.0.0D0)VRED=8.5D0
       IF(TINC.EQ.0.0D0)TINC=0.5D0
       IF(IDR1.EQ.0)IDR1=2
       IF(IDR2.EQ.0)IDR2=128
       IF(ITEST.EQ.0)ITEST=100
       WRITE(6,104)RE,R1,R3,R2,IDR1,IDR2,ITEST
     3 CONTINUE
C
C      DATA CARD SET 1 + 5 2F10.0,I5,54X,I1  AI1,AI2,NRAYS,ISTOP
C      IF ISTOP = 9 , TERMINATE PROGRAM
C      IF ISTOP = 8 , RE-ENTER PROGRAM AT DATA CARD SET 1 + 4
C      IF ISTOP = 7 , RE-ENTER PROGRAM AT DATA CARD SET 1
C      AI1 IS MAXIMUM ANGLE OF INCIDENCE (DEGREES)
C      AI2 IS MINIMUM ANGLE OF INCIDENCE (DEGREES)
C      IF AI1 IS UNSPECIFIED, AI1 = 89.5, AI2 = 20.0
C      NRAYS IS NUMBER OF RAYS PASSED PER SHELL FOR FLUX CALCULATION
C      NRAYS = NRAY IF NRAYS.LE.NRAY AND RAY FLUX IS DELETED
C      *****RAY FLUX CALCULATED ONLY IF NRAYS.GT.NRAY*********
C
       READ(5,108)AI1,AI2,NRAYS,ISTOP
       IF(ISTOP.EQ.9)CALL EXIT
       IF(ISTOP.EQ.8)GO TO 1
       IF(ISTOP.EQ.7)GO TO 2
       IF(AI1.GT.90.0D0)AI1=90.0D0
       IF(AI1)1010,1010,1011
  1010 CONTINUE
       AI1 = 89.5D0
       AI2=20.0D0
  1011 CONTINUE
       AI1=AI1*RADDEG
       AI2=AI2*RADDEG
       IF(NRAYS.LT.NRAY)NRAYS=NRAY
       CALL SET(RDEL,VDEL,J$,I$,RMAN,VMAN,K$,RO,VO,L$,1,IDIMO)
       IDIMOM = IDIMO - 1
```

```
      INTR = 1
      DO 1000 I = 2, IDIMOM
      AI=(RO(I)*VO(1))/(VO(I)*RO(1))
      IF(AI.GT.1.0D0)GO TO 1000
      AI=DARSIN(AI)
      IF(AI.GT.AI1)GO TO 1000
      IF(AI.LT.AI2)AI=AI2
      AINC=(AI1-AI)/DFLOAT(NRAYS-1)
      DAI=0.10D0*RADDEG
      IF(AINC.LT.DAI)DAI=AINC
      IF(NRAYS.EQ.NRAY)DAI=0.0D0
      INTRVL = I - 1
      IF (INTR .NE. INTRVL) GO TO 1001
      ANGLE1=AI1/RADDEG
      ANGLE2=AI/RADDEG
      WRITE(6,110)ANGLE1,ANGLE2,INTRVL
      DO 1001 II=1,NRAYS
      IA=II-1
      AI=AI1-AINC*DFLOAT(IA)
      AI=AI/RADDEG
      WRITE(6,103)AI
      AI=AI1-AINC*DFLOAT(IA)
      IF(II.EQ.1)AI=AI-1.0D-8
      IF(II.EQ.NRAYS)AI=AI+1.0D-8
      IOUT=IFIX(XOUT)
     OCALL RAY (RDEL,VDEL,DELMOD,J$,I$,RMAN,VMAN,K$,R,V,L$,AI,R1,R2,R3,
     1VRED,TINC,IOUT,IDR1,IDR2,ITEST,RADDEG,DELTA,TIME,RE,P)
      DELTAC=DELTA
      TIMEC=TIME
      IF(DAI.EQ.0.0D0)GO TO 5
      IC=1
     OCALL D VEL(R,V,L$,
     1R3,V1,IC,IDIM)
C
C     V1=ANGLE OF EMERGENCE
C
C
      V1=DARSIN(P*V1/R3)
      DELTAC=DELTA
      AI=AI1-AINC*DFLOAT(IA)-DAI
      IOUT=0
     OCALL RAY (RDEL,VDEL,DELMOD,J$,I$,PMAN,VMAN,K$,R,V,L$,AI,R1,R2,R3,
     1VRED,TINC,IOUT,IDR1,IDR2,ITEST,RADDEG,DELTA,TIME,RE,P)
      DELTAM=DELTA
      IF(II.EQ.1)GO TO 1012
      AI=AI1-AINC*DFLOAT(IA)+DAI
     OCALL RAY (RDEL,VDEL,DELMOD,J$,I$,RMAN,VMAN,K$,R,V,L$,AI,R1,R2,R3,
     1VRED,TINC,IOUT,IDR1,IDR2,ITEST,RADDEG,DELTA,TIME,RE,P)
 1012 CONTINUE
      DELTAP=DELTA
      EP=DABS(DAI*RADDEG)*DSIN(AI1*RADDEG)/DCOS(V1)
      EM=EP/(DSIN((DELTAC+DELTAM)/2.0D0)*DABS(DELTAC-DELTAM))
      EP=EP/(DSIN((DELTAC+DELTAP)/2.0D0)*DABS(DELTAC-DELTAP))
      EMLOG=EM
      EMLOG=ALOG10(EMLOG)
```

```
      EPLOG=EP
      EPLOG=ALOG10(EPLOG)
      AI=AI1-AINC*DFLOAT(IA)
      DIST=DELTAC*RE
      TRED=(TIMEC-DELTAC*RE/VRED)
      IF(II.EQ.1)EPLOG=EMLOG
      IF(II.EQ.NRAYS)EMLOG=EPLOG
      FLUX=(EPLOG+EMLOG)/2.0
      WRITE(6,105)EP,EPLOG,EM,EMLOG,FLUX
      MODEL1=1
    5 CONTINUE
 1001 CONTINUE
      INTR = INTRVL +1
      AI1=AI
 1000 CONTINUE
      GO TO 3
      END
```

```
      OSUBROUTINE RAY(RDEL,VDEL,DELMOD,J$,I$,RMAN,VMAN,K$,R,V,L$,
     1AI,R1,R2,R3,VRED,TINC,IOUT,IDR1,IDR2,ITEST,RADDEG,DELTA,TIME,RE,P)
C
C      CALCULATES TIME, DISTANCE AND RAY PATH FOR A RAY OF GIVEN ANGLE
C      OF INCIDENCE
C
       REAL*4 RDEL(J$,I$),VDEL(J$,I$),DELMOD(J$),RMAN(K$),VMAN(K$)
       REAL*8 R(L$),V(L$)
       REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
       REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
       REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL,REND
 1000 FORMAT('0',5X,'DELTA =',F8.2,'KM  T-DELTA/',F4.2,' = ',F7.2,
      1' SEC   RAD BTM =',F7.2,'KM')
 1020 FORMAT('+',78X,'DELTA =',F8.2,'KM  T-DELTA/',F4.2,' = ',
      1F7.2,' SEC')
       MODEL1=1
      OIF(MODEL.NE.MODEL1)CALL SET(RDEL,VDEL,J$,I$,RMAN,VMAN,K$,R,V,L$,
      1MODEL1,IDIM)
       MODEL=1
       P=0.0D0
       DELTA=0.0D0
       TIME=0.0D0
       THOLD=0.0D0
       IF(AI.GE.90.0D0*RADDEG)RETURN
      OCALL DOWN I(R,V,L$,
      1AI,R1,R2,R3,IDR1,IDR2,ITEST,IDIM,IOUT,DELTA,TIME,RE,P,RAD,THOLD,
      2TINC,IDR,IREFLT,REND)
       DELTAK=RE*DELTA
       DELTAD=DELTA/RADDEG
       TRED=TIME-DELTAK/VRED
        WRITE(6,100) DELTAK,VRED,TRED,RAD
       DELTAK=DELTAK+DELTAK
       IC=1
      OCALL D VEL(R,V,L$,
      1REND,PE2,IC,IDIM)
       IF(IREFLT.NE.0)PE2=DARSIN(P*PE2/REND)
      OCALL MOD(DELMOD,J$,
      1DELTAK,MODEL)
      OIF(MODEL.NE.MODEL1)CALL SET(RDEL,VDEL,J$,I$,RMAN,VMAN,K$,R,V,L$,
      1MODEL,IDIM)
    2 CONTINUE
       IF(MODEL.EQ.MODEL1)GO TO 5
       IC=1
      OCALL D VEL(R,V,L$,
      1REND,V1,IC,IDIM)
       IF(IREFLT.EQ.0)P=REND/V1
       IF(IREFLT.NE.0)P=(REND/V1)*DSIN(PE2)
    5 CONTINUE
       IDR=1
       DO 3 I=1,IDR2
       IDR=IDR*2
       IF(IDR.GE.IDR2)GO TO 4
    3 CONTINUE
    4 CONTINUE
```

```
1 CONTINUE
  0CALL UP I(R,V,L$,
 1AI,R1,R2,R3,IDR1,IDR2,ITEST,IDIM,IOUT,DELTA,TIME,RE,P,RAD,THOLD,
 2TINC,IDR,IREFLT,REND)
   IDR=IDR1
   RAD=P1
   DELTAK=RE*DELTA
   DELTAD=DELTA/RADDEG
   TRED=TIME-DELTAK/VRED
   WRITE(6,102) DELTAK,VRED,TRED
   RETURN
   END
```

```
     OSUBROUTINE DOWN I(R,V,L$,
     1AI,R1,R2,R3,IDR1,IDR2,ITEST,IDIM,IOUT,DELTA,TIME,RE,P,RAD,THOLD,
     2TINC,IDR,IREFLT,REND)
C
C      INTEGRATES FROM R1 TO REND (RAY BOTTOM)
C      USING CENTER POINT INTEGRATION FORMULA
C
       REAL*8 R(L$),V(L$)
       REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
       REAL*8 DELTAC,DELTAM,DELTAP,EM,FP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
       REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL,REND
  100 FORMAT(1X,5(7X,'X',7X,'Z',7X,'T'))
  101 FORMAT(8(1X,D14.8))
       IC=1
       ID=0
       IF(P1.EQ.0.0D0)P1=R(1)
      OCALL D VEL(R,V,L$,
      1R1,V1,IC,IDIM)
       IF(P.EQ.0.0D0)P=(R1/V1)*DSIN(AI)
       PE2=P*P
       CALL BOTTOM(R,V,IDIM,P,AI,REND,IREFLT)
       RAD=R1
       IT=IC
       TEST=DFLOAT(ITEST)
       IDR=IDR1
       DR=1/DFLOAT(IDR)
       IF(IOUT.EQ.1)WRITE(6,100)
    4 CONTINUE
       RADC=RAD-DR/2.0D0
       RADT=RAD-DR*TEST
      OCALL D VEL(R,V,L$,
      1RADC,VELC,IC,IDIM)
      OCALL D VEL(R,V,L$,
      1RADT,VELT,IT,IDIM)
       IF(RADT/VELT.LE.P)GO TO 1
       IF(RADT/VELC.LE.P)GO TO 2
       IF(RAD-DR.LT.R2)GO TO 3
       RDV=RADC/VELC
       A=RDV*RDV-PE2
       A=DR/DSQRT(RDV*RDV-PE2)
       DELTA=DELTA+P*A/RADC
       TIME=TIME+RDV*A/VELC
       RAD=RAD-DR
       IF(IOUT.EQ.1)CALL OUT(DELTA,TIME,RE,TINC,THOLD,RAD,ID,0)
       GO TO 4
    1 CONTINUE
    2 CONTINUE
       IT=1
       IDR=IDR*2
       DR=1/DFLOAT(IDR)
       IF(TEST.EQ.1.0D0)GO TO 5
       IF(IDR.GE.IDR2)TEST=1.0D0
       GO TO 4
    5 CONTINUE
```

```
      IF(IOUT.EQ.1)CALL OUT(DELTA,TIME,RE,TINC,THOLD,RAD,ID,1)
      RETURN
    3 CONTINUE
C
C     MODIFY LAST STEP IN INTEGRATION TO THIN HOMOGENEOUS SHELL FORMULA
C     USE THIS FORMULA ONLY IF IREFLT =0
C     OTHERWISE RETAIN PRESENT FORM
C
      DR=RAD-R2
      RADC=RAD-DR/2.0D0
     0CALL DVEL(R,V,L$,
     1RADC,VELC,IC,IDIM)
      RDV=RADC/VELC
      IF(RDV*RDV-PE2.LE.0.0D0)GO TO 6
      A=DR/DSQRT(RDV*RDV-PE2)
      DELTA=DELTA+P*A/RADC
      TIME=TIME+RDV*A/VELC
      RAD=RAD-DR
    6 CONTINUE
      IF(IOUT.EQ.1)CALL OUT(DELTA,TIME,RE,TINC,THOLD,RAD,ID,1)
      RETURN
      END
```

```
      OSUBROUTINE UP I(R,V,L$,
     1AI,R1,R2,R3,IDR1,IDR2,ITEST,IDIM,IOUT,DELTA,TIME,RE,P,RAD,THOLD,
     2TINC,IDR,IREFLT,REND)
C
C        INTEGRATES FROM REND (RAY BOTTOM) TO R3
C        USING CENTER POINT INTEGRATION FORMULA
C
      REAL*8 R(L$),V(L$)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL,REND
  100 FORMAT(1X,5(7X,'X',7X,'Z',7X,'T'))
  101 FORMAT(8(1X,D14.8))
      IC=1
      ID=0
      OCALL U VEL(R,V,L$,
     1RAD,VEL,IC,IDIM)
      IF(P.EQ.0.0D0)P=(RAD/VEL)*DSIN(AI)
      PE2=P*P
      IF(R3.EQ.0.0D0)R3=R(1)
      IT=1
      TEST=DFLOAT(ITEST)
      DR=1/DFLOAT(IDR)
      IF(IOUT.EQ.1)WRITE(6,100)
C
C        MODIFY FIRST STEP IN INTEGRATION TO THIN HOMOGENEOUS SHELL FORMULA
C        USE THIS FORMULA ONLY IF IREFLT =0
C        OTHERWISE RETAIN PRESENT FORM
C
    4 CONTINUE
      RADC=RAD+DR/2.0D0
      RADT=RAD-DR*TEST
      OCALL U VEL(R,V,L$,
     1RADC,VELC,IC,IDIM)
      OCALL U VEL(R,V,L$,
     1RADT,VELT,IT,IDIM)
      IF(RADT/VELT.GT.P)GO TO 1
      IF(RADT/VELC.GT.P)GO TO 2
      IF(RAD+DR.GE.R3)GO TO 3
      RDV=RADC/VELC
      A=RDV*RDV-PE2
      A=DR/DSQRT(RDV*RDV-PE2)
      DELTA=DELTA+P*A/RADC
      TIME=TIME+RDV*A/VELC
      RAD=RAD+DR
      IF(IOUT.EQ.1)CALL OUT(DELTA,TIME,RE,TINC,THOLD,RAD,ID,0)
      GO TO 4
    1 CONTINUE
    2 CONTINUE
      IDR=IDR/2
      DR=1/DFLOAT(IDR)
      IT=1
      IF(IDR.GT.IDR1)GO TO 4
      DR=1/DFLOAT(IDR1)
```

```
      TEST=RAD/DR
      GO TO 4
3  CONTINUE
      DR=P3-RAD
      RADC=RAD+DR/2.0D0
   0CALL U VEL(R,V,L$,
   1RADC,VELC,IC,IDIM)
      RDV=RADC/VELC
      IF(RDV*RDV-PE2.LE.0.0D0)GO TO 5
      A=DR/DSQRT(RDV*RDV-PE2)
      DELTA=DELTA+P*A/RADC
      TIME=TIME+RDV*A/VELC
      RAD=RAD+DR
5  CONTINUE
      IF(IOUT.EQ.1)CALL OUT(DELTA,TIME,RE,TINC,THOLD,RAD,ID,1)
      RETURN
      END
```

```
      SUBROUTINE DOWN I(R,V,L$,
     1AI,R1,R2,R3,IDR1,IDR2,ITEST,IDIM,IOUT,DELTA,TIME,RE,P,RAD,THOLD,
     2TINC,IDR,IREFLT,REND)
C
C      INTEGRATES FROM R1 TO REND (RAY BOTTOM)
C      USING THIN HOMOGENEOUS SHELL FORMULA
C
      REAL*8 R(L$),V(L$)
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,PAD,DELTAK,DELTAD,TRFD
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 RHOLD,REND,DRD2,RADMDR,RADPDR,PVC,PVCE2
      IC = 1
      ID = 0
      IDR2D2=IDR2/2
      DELTA = 0.0D0
      TIME = 0.0D0
      IF(R1 .LE. 0.0D0) R1 = R(1)
      RHOLD =R1
      CALL D VEL (R,V,L$,R1,V1,IC,IDIM)
      IF ( P .EQ. 0.0D0) P = R1*DSIN(AI)/V1
      PE2 = P*P
      CALL BOTTOM (R,V,IDIM,P,AI,REND,IREFLT)
      TEST = DFLOAT(ITEST)
      IF(IREFLT .EQ. 1) TEST = C.0D0
      RAD = R1
      IDR = IDR1
    3 CONTINUE
      DR = 1.0D0/DFLOAT(IDR)
      DRD2 = DR/2.0D0
      I = 0
    1 CONTINUE
      I = I + 1
      RADC = RAD -DRD2
      RADMDR =RHOLD - FLCAT(I)*DR
      IF(RADMDR -TEST*DR .LE. REND) GO TO 4
      CALL D VEL (R,V,L$,RADC,VELC,IC,IDIM)
      PVC = P * VELC
      PVCE2 = PVC * PVC
      DELTA = DELTA+DARCOS(PVC/RAD)-DARCOS(PVC/RADMDR)
      TIME = TIME+(DSGRT(RAD*RAD-PVCE2)-DSGRT(RADMDR*RADMDR-PVCE2))/VELC
      RAD = RADMDR
      GO TO 1
    4 CONTINUE
      IF (IREFLT .EQ. 1) GO TO 2
      RHOLD = RAD
      I = 0
      IDR = IDR*2
      IF(IDR .LE. IDR2D2)TEST=0.0D0
      IF (IDR .LE. IDR2) GO TO 3
    2 CONTINUE
      DR =(RAD -REND)/2.0D0
      DRD2=DR /2.0D0
      RADC = RAD - DRD2
```

```
      CALL D VEL (R,V,LS,RADC,VELC,IC,IDIM)
      PVC = P* VELC
      PVCE2=PVC * PVC
      DELTA=DELTA+DARCOS(PVC/RAD)
      TIME=TIME+(DSQRT(RAD*RAD-PVCE2))/VELC
      IF(IREFLT.EQ.0)GO TO 5
      DELTA=DELTA-DARCOS(PVC/REND)
      TIME=TIME-DSQRT(REND*REND-PVCE2)/VELC
    5 CONTINUE
      RAD = REND
      RETURN
      END
```

```
      SUBROUTINE UP I(R,V,L$,
     1AI,R1,R2,R3,IDR1,IDR2,ITEST,IDIM,IOUT,DELTA,TIME,RE,P,RAD,THOLD,
     2TINC,IDR,IREFLT,REND)
C
C     INTEGRATES FROM REND (RAY BOTTOM) TO R3
C     USING THIN HOMOGENEOUS SHELL FORMULA
C
C
      REAL*8 R(L$),V(L$)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,FP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
      REAL*8 RHOLD,REND,DRD2,RADMDR,RADPDR,PVC,PVCE2
      TEST = DFLOAT(ITEST)
      IC = 1
      ID = 0
      IDR=IDR2
      DR=1.0D0/DFLOAT(IDR)
      DRD2=DR/2.0D0
      RADC=RAD+DRD2
      CALL U VEL (R,V,L$,RADC,VELC,IC,IDIM)
      IF(PVC/RAD.LT.1.0D0)GO TO 5
      RADPDR=RAD+DR
      PVC=P*VELC
      PVCE2=PVC*PVC
      DELTA=DELTA+DARCOS(PVC/RADPDR)
      TIME=TIME+(DSQRT(RADPDR*RADPDR-PVCE2))/VELC
      RAD=RADPDR
    5 CONTINUE
      CALL U VEL(R,V,L$,RAD,VEL,IC,IDIM)
      IF(P.NE.0.0D0)GO TO 4
      P=RAD*DSIN(AI)/VEL
      IREFLT=2
    4 CONTINUE
      IF(IREFLT.EQ.0)P=RAD/VEL
      IF(RAD/VEL.LT.P)P=RAD/VEL
      PE2 = P*P
      IF (R3 .LE. 0.0D0) R3 = R(1)
      I = 0
    3 CONTINUE
      RHOLD = RAD
      DR = 1.0D0/DFLOAT(IDR)
      DRD2 = DR/2.0D0
    2 CONTINUE
      RADC = RAD + DRD2
      I = I + 1
      RADPDR = RHOLD + FLOAT(I)*DR
      IF (RADPDR .GE. R3) GO TO 1
      CALL U VEL(R,V,L$,RADC,VELC,IC,IDIM)
      PVC = P* VELC
      PVCE2 = PVC * PVC
      DELTA =DELTA+DARCOS(PVC/RADPDR)-DARCOS(PVC/RAD)
      TIME=TIME+(DSQRT(RADPDR*RADPDR-PVCE2)-DSQRT(RAD*RAD-PVCE2))/VELC
      RAD = RADPDR
      IF (RAD -TEST*DR .GT. REND) GO TO 2
```

```
      IDR = IDR/2
      I = 0
      IF(IDR .GT. IDR1) GO TO 3
      IDR = IDR1
      DR = 1.0D0/DFLOAT(IDR)
      TEST = RAD/DR
      GO TO 3
    1 CONTINUE
      IDR = IDR1
      DR = R3 - RAD
      DRD2 = DR/2.0D0
      RADC =(R3+RAD)/2.0D0
      CALL U VEL (R,V,L$,RADC,VELC,IC,IDIM)
      PVC = P* VELC
      PVCE2 = PVC*PVC
      DELTA = DELTA+(DARCOS(PVC/R3)-DARCOS(PVC/RAD))
      TIME = TIME+(DSQRT(R3*R3-PVCE2)-DSQRT(RAD*RAD-PVCE2))
      RAD = R3
      IDR = IDR1
      RETURN
      END
```

```
      OSUBROUTINE UP I(R,V,L$,
     1AI,R1,R2,R3,IDR1,IDR2,ITEST,IDIM,IOUT,DELTA,TIME,RE,P,RAD,THOLD,
     2TINC,IDR,IREFLT,REND)
C
C     INTEGRATES FROM REND (RAY BOTTOM) TO R3
C     ****ASSUMING    R3 = R1
C     ****ASSUMING    SAME VELOCITY STRUCTURE AS USED IN DOWN I
C     *******NOTE*********    THIS ROUTINE DOES NOT CALCULATE RAY PATH ***
C
      REAL*8 R(L$),V(L$)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL,REND
  100 FORMAT(1X,5(7X,'X',7X,'Z',7X,'T'))
  101 FORMAT(8(1X,D14.8))
      DELTA=DELTA+DELTA
      TIME=TIME+TIME
      RAD=R1
      IDR=IDR1
      RETURN
      END
```

```
0SUBROUTINE D VEL(R,V,L$,
1RAD,VEL,IC,IDIM)
C
C    CALCULATES VELOCITY AT A GIVEN RADIUS FOR DECREASING R
C    USING LINEAR INTERPOLATION FORMULA BETWEEN V(I), V(I+1)
C    WHERE    R(I).GT.R.GE.R(I+1)
C
     REAL*8 R(L$),V(L$)
     REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
     REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
     REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
     IF(RAD.GE.R(1))GO TO 1
     ISTART=IC
     IDIM1=IDIM-1
     DO 2 I=ISTART,IDIM1
     IC=I
     IP1=I+1
     IF(RAD.GE.R(IP1))GO TO 3
2 CONTINUE
3 CONTINUE
     VEL=((V(IP1)-V(I))*RAD+V(I)*R(IP1)-V(IP1)*R(I))/(R(IP1)-R(I))
     RETURN
1 CONTINUE
     VEL=V(1)
     IC=1
     RETURN
     END
```

```
      OSUBROUTINE U VEL(R,V,L$,
     1RAD,VEL,IC,IDIM)
C
C     CALCULATES VELOCITY AT A GIVEN RADIUS FOR INCREASING R
C     USING LINEAR INTERPOLATION FORMULA BETWEEN V(I), V(I+1)
C     WHERE    R(I).GT.R.GE.R(I+1)
C
      REAL*8 R(L$),V(L$)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
      ISTART=IC
      DO 1 I=ISTART,IDIM
      J=IDIM-I+1
      IC=I
      IF(RAD.LT.R(J))GO TO 2
    1 CONTINUE
    2 CONTINUE
      IF(RAD.GT.R(1))GO TO 3
      I=J
      IP1=J+1
      VEL=((V(IP1)-V(I))*RAD+V(I)*R(IP1)-V(IP1)*R(I))/(R(IP1)-R(I))
      RETURN
    3 CONTINUE
      VEL=V(1)
      RETURN
      END
```

```
      OSUBROUTINE SET(RDEL,VDEL,J$,I$,RMAN,VMAN,K$,R,V,L$,
     1MODEL,IDIM)
C
C     SETS R,V TO RDEL,VDEL,RMAN,VMAN ASSUMING NUMBER 'MODEL'
C
      REAL*8 R(L$),V(L$)
      REAL*4 RDEL(J$,I$),VDEL(J$,I$),RMAN(K$),VMAN(K$)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
      DO 1 I=1,I$
      IF(RDEL(MODEL,I).LE.0.0)GO TO 2
      II=I
      R(II)=DFLOAT(IFIX((RDEL(MODEL,II)*1.0E2)+0.5))/1.0D2
      V(II)=DFLOAT(IFIX((VDEL(MODEL,II)*1.0E4)+0.5))/1.0D4
    1 CONTINUE
    2 CONTINUE
      II=II+1
      L$M1=L$-1
      DO 3 I=II,L$M1
      J=I-II+1
      IF(RMAN(J).LE.0.0)GO TO 4
      IDIM=I
      R(I)=DFLOAT(IFIX((RMAN(J)*1.0E2)+0.5))/1.0D2
      V(I)=DFLOAT(IFIX((VMAN(J)*1.0E4)+0.5))/1.0D4
    3 CONTINUE
    4 CONTINUE
      IDIM=IDIM+1
      R(IDIM)=0.0D0
      V(IDIM)=V(IDIM-1)
  100 FORMAT(1X,10F13.5)
      RETURN
      END
```

```
      OSUBROUTINE MOD(DELMOD,J$,
     1DELTAK,MODEL)
C
C     CALCULATES MODEL NUMBER (MODEL) FOR A GIVEN RANGE FROM SOURCE
C
      REAL*8 RADDEG,RE,R1,R2,R3,VREC,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TPED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
      REAL*4 DELMOD(J$)
      DO 1 I=1,J$
      MODEL=I
      IF(DELTAK.LE.DELMOD(I))RETURN
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE OUT(DELTA,TIME,RE,TINC,THOLD,RAD,ID,IDUMP)
C
C     OUTPUTS RAY PATH CO-ORDINATES X,Z,T
C
C
C     MODIFY FOR CALCOMP PLOT OUTPUT
C
      REAL*4 X(5),Z(5),T(5)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
  100 FORMAT(1X,5(2F8.1,F8.2))
      IF(IDUMP.EQ.1)GO TO 1
      IF(TIME-THOLD.LT.TINC)RETURN
      ID=ID+1
      X(ID)=RAD*DSIN(DELTA)
      Z(ID)=RE-RAD*DCOS(DELTA)
      T(ID)=TIME
      THOLD=THOLD+TINC
      IF(ID.LT.5)RETURN
      WRITE(6,100)((X(K),Z(K),T(K)),K=1,ID)
      ID=0
      RETURN
    1 CONTINUE
      ID=ID+1
      X(ID)=RAD*DSIN(DELTA)
      Z(ID)=RE-RAD*DCOS(DELTA)
      T(ID)=TIME
      WRITE(6,100)((X(K),Z(K),T(K)),K=1,ID)
      ID=0
      RETURN
      END
```

```
      SUBROUTINE DATA(RDEL,VDEL,DELMOD,J$,I$,RMAN,VMAN,K$,R,V,L$)
C
C     READS VELOCITY STRUCTURE  DATA CARDS SET 1
C
      REAL*8 R(L$),V(L$)
      REAL*4 RDEL(J$,I$),VDEL(J$,I$),DELMOD(J$),RMAN(K$),VMAN(K$)
      REAL*8 RADDEG,RE,R1,R2,R3,VRED,TINC,AI1,AI2,AINC,DAI,AI,DELTA,TIME
      REAL*8 DELTAC,DELTAM,DELTAP,EM,EP,SLOPE,P,RAD,DELTAK,DELTAD,TRED
      REAL*8 V1,PE2,TEST,DR,RADC,RADT,VELC,VELT,RDV,A,THOLD,VEL
 1000FORMAT('1RAY PATHS & TRAVEL TIME CURVE FOR A SPHERICALLY SYMMETRIC
     1 EARTH')
 1010FORMAT(1X,'
     1                               ',I1)
  102 FORMAT(8F10.0)
  103 FORMAT('0',5(6X,'RADIUS',4X,'VELOCITY'))
  104 FORMAT(1X,10F12.5)
  105 FORMAT(' MODEL',I4,'  USED FOR DELTA LESS THAN',F10.1,'KM')
  106 FORMAT(' MODEL',I4,'  USED FOR DELTA LESS THAN',F14.8,'KM')
  107 FORMAT('1MODEL',I5)
      WRITE(6,100)
      DO 1 J=1,50
C
C     DATA CARDS SET 1.1  TERMINATETED WITH 9 IN COLUMN 80
C     THESE CARDS CONTAIN THE TITLE TO THE CRUSTAL VELOCITY STRUCTURE
C     EACH CARD IS REPRODUCED IN THE OUTPUT
C
    2 CONTINUE
      READ(5,101)ISTOP
      WRITE(6,101)
      IF(ISTOP.NE.9)GO TO 2
C
C     DATA CARDS SET 1.2(1 CARD) F10.0 DELMOD(J) J=1 INITIALLY
C     AZIMUTH RANGE (KM) OF MODEL J LIES BETWEEN DELMOD(J-1),DELMOD(J)
C
      READ(5,102)DELMOD(J)
      WRITE(6,105)J,DELMOD(J)
      DO 3 I=1,20,4
      N=I+3
C
C     DATA CARDS SET 1.3 8F10.0 RDEL,VDEL OF MODEL J
C     TERMINATED BY READING 5 CARDS OR LAST RDEL OF CARD LE.0.0
C     IF RMAN (LAST OF CARD) IS LT 0.0 GO ON TO DATA CARDS SET 1.4
C     IF RDEL (LAST OF CARD) IS 0.0,READ NEXT DELMOD AND RMAN,VMAN
C     BY READING NEXT SET OF DATA CARDS SET 1.1, 1.2, AND 1.3
C     THIS CAN BE DONE FOR 50 MODELS AT MOST
C
      READ(5,102)((RDEL(J,K),VDEL(J,K)),K=I,N)
      IF(RDEL(J,N).LE.0.0)GO TO 4
    3 CONTINUE
    4 CONTINUE
      WRITE(6,103)
      WRITE(6,104)((RDEL(J,K),VDEL(J,K)),K=1,N)
      NMODEL=J
      IF(RDEL(J,N).LT.0.0)GO TO 5
```

```
      1 CONTINUE
      5 CONTINUE
        DELMOD(NMODEL)=1.0E70
        WRITE(6,106)NMODEL,DELMOD(NMODEL)
C
C       DATA CARDS SET 1.4 TERMINATED WITH 9 IN COLUMN 80
C       TITLE OF MANTLE VELOCITY STRUCTURE
C
      6 CONTINUE
        READ(5,101)ISTOP
        WRITE(6,101)
        IF(ISTOP.NE.9)GO TO 6
        DO 7 I=1,80,4
        N=I+3
C
C       DATA CARDS SET 1.5 8F10.0 RMAN,VMAN
C       TERMINATED BY READING 10 CARDS OR LAST RMAN OF A  CARD LE 0.0
C
        READ(5,102)((RMAN(K),VMAN(K)),K=I,N)
        IF(RMAN(N).LE.0.0)GO TO 8
      7 CONTINUE
      8 CONTINUE
        N=N-4
        DO 9 I=1,4
        IN=I+N
        IF(RMAN(I).LE.0.0)GO TO 10
      9 CONTINUE
     10 CONTINUE
        RMAN(IN)=0.0
        IF(VMAN(IN).LE.0.0)VMAN(IN)=VMAN(IN-1)
        WRITE(6,103)
        WRITE(6,104)((RMAN(K),VMAN(K)),K=1,IN)
        DO 11 I=1,NMODEL
       CCALL SET(RDEL,VDEL,J$,I$,RMAN,VMAN,K$,R,V,L$,
      1I,IDIM)
        WRITE(6,107)I
        WRITE(6,103)
        WRITE(6,104)((R(K),V(K)),K=1,IDIM)
     11 CONTINUE
        RETURN
        END
```

```
      SUBROUTINE BOTTOM(R,V,IDIM,P,AI,REND,IRFFLT)
C
C     CALCULATES THE MINIMUM RADIUS (REND) OF A GIVEN RAY
C
C
C     IRFFLT = 0 IN CASE OF REFRACTED RAY
C     IREFLT = 1 IN CASE OF REFLECTED RAY
C
      REAL*8 R(IDIM), V(IDIM)
      REAL *8 P,AI,REND,A,B,C
      IF (P) 3,2,3
    2 CONTINUE
      P= (R(1)*DSIN(AI))/V(I)
    3 CONTINUE
      IREFLT = 0
      IDIMM1 = IDIM - 1
      DO 1 I = 1,IDIMM1
      IP1 = I + 1
      IF(R(IP1) .EQ. R(I)) GO TO 1
      B = R(IP1) - R(I)
      A = (V(IP1) - V(I))/B
      B =(R(IP1) * V(I) - R(I)*V(IP1))/B
      REND =(B*P)/(1.0D0-(A*P))
      IF (REND .GE. R(IP1)) GO TO 5
    1 CONTINUE
    5 CONTINUE
      IF (REND .LT. R(I)) GO TO 6
      IREFLT = 1
      REND = R(I)
    6 CONTINUE
      RETURN
      END
```

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